Lecture: Financial Modelling

– Overview

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Outline

- Finance is the study of how and under what terms savings (money) are allocated between lenders and borrowers.
  - Finance is distinct from economics in that, it addresses not only how resources are allocated but also under what terms and through what channels.
- Financial contracts or securities occur whenever funds are transferred from issuer to buyer.

- **Real assets** are *tangible* that are owned by persons and businesses:
  - Residential structures and property;
  - Major appliances and automobiles;
  - Office towers, factories, mines;
  - Machinery and equipment.

- **Financial assets** are what one individual has lent to another:
  - Consumer credit;
  - Loans;
  - Mortgages.

- Financial modeling is the task of building an abstract representation (a model) of a real world financial situation.
  - This is a mathematical model designed to represent (a simplified version of) the performance of a financial asset or portfolio of a business, project, or any other investment.
  - Financial modeling is an exercise in either asset pricing, risk management or corporate finance, of a quantitative nature.
  - Financial modeling is about translating a set of hypotheses about the behavior of markets or agents into numerical predictions.

- Security pricing
  - Primary securities: stocks, bonds, ...
  - Derivative securities: forwards, futures, swaps; options, ...

- Portfolio selection: choose a trading strategy to maximize the utility of consumption and final wealth.
  - Related to security pricing;
  - Single-period models: Markowitz portfolio selection;
  - Continuous-time models: dynamic portfolio selection.

- Risk management: understand the risks inherent in a portfolio
  - Tail risk: probability of large losses;
  - Value-at-risk (VaR) and conditional value-at-risk (CVaR);
  - Starting to become important for portfolio selection as well.

- The household is the primary provider of funds to businesses and government:
  - Households must accumulate financial resources throughout their working life times to have enough savings (pension) to live on in their retirement years.
- Financial intermediaries transform the nature of the securities they issue and invest in:
  - Banks, trust companies, credit unions, insurance firms, mutual funds.
- Market intermediaries simply help make markets work:
  - Investment dealers;
  - Brokers.

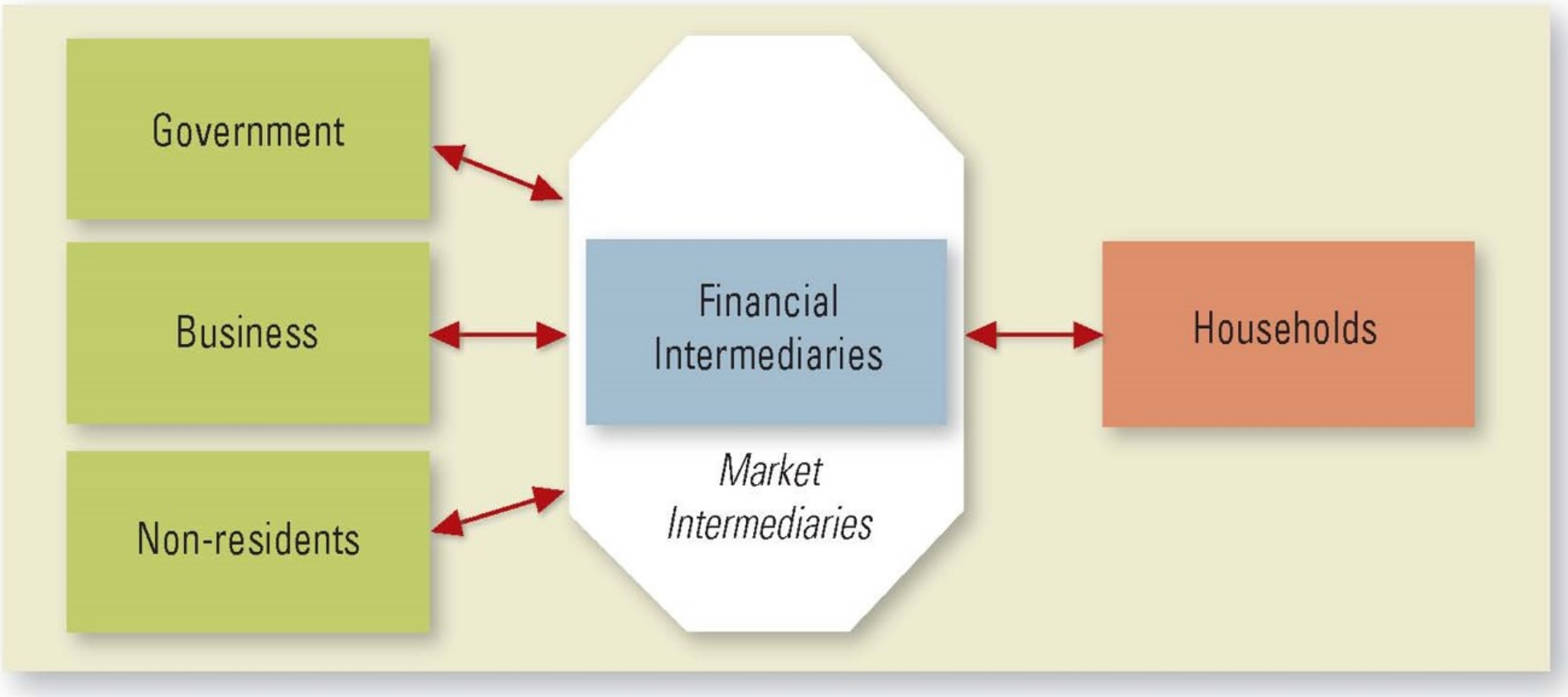


Figure: Financial system

- Funds can be channeled from the saver to borrower in three ways:
  1. Direct intermediation: direct transfer from saver to borrower, i.e., a non-market transaction.
  2. Direct intermediation: a market-based transaction usually through a market intermediary, such as a broker.
  3. Indirect claims through a financial intermediary: such as a bank who offers deposit-taking services and ultimately lends those deposits out as mortgages or loans.

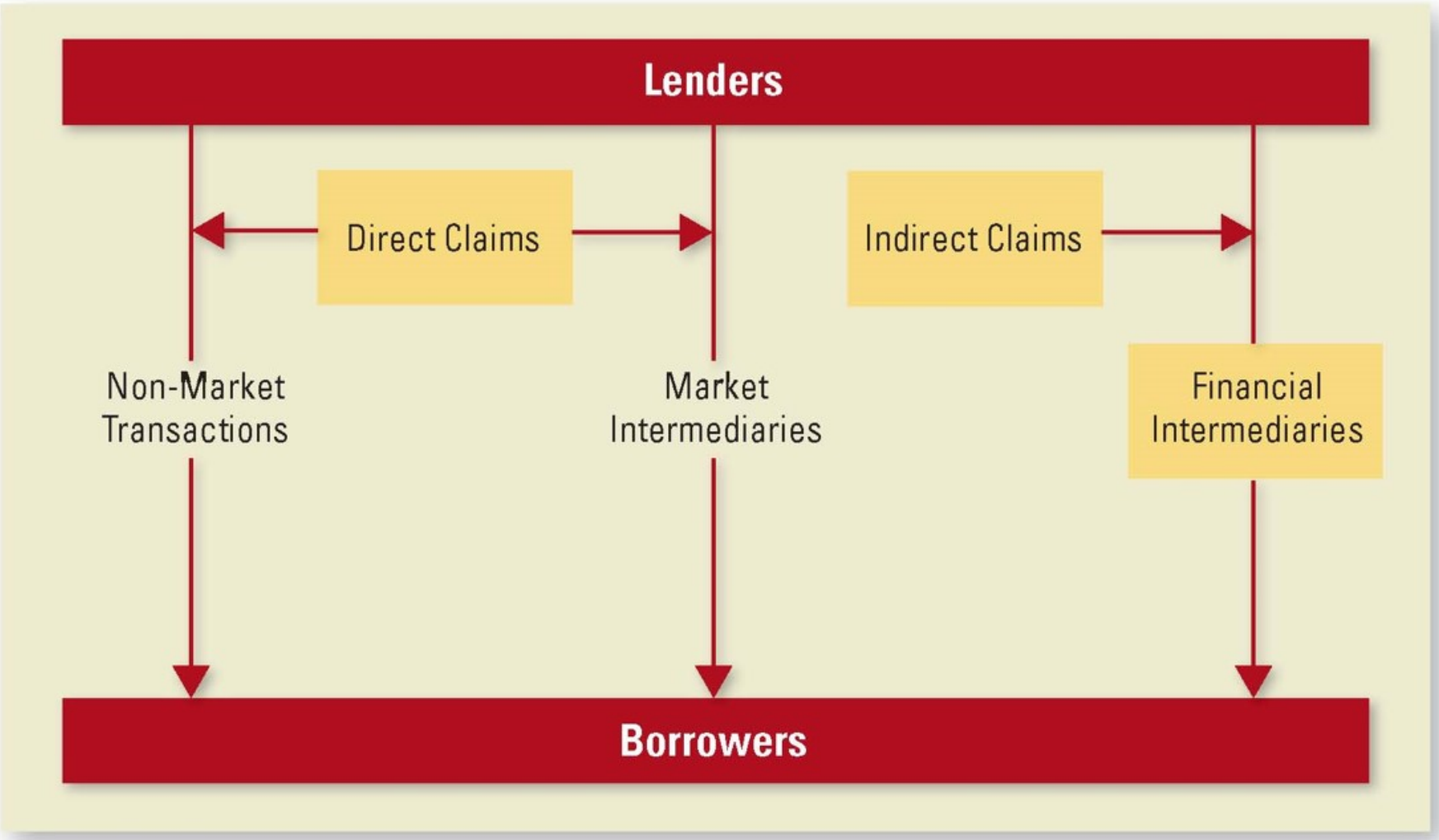


Figure: Channels of intermediation

Financial markets can be classified by the time-to-maturity:

1. **Money market securities** (for short-term debt securities that are pure discount notes):
   - Commercial paper (CP);
   - Treasury bills.

2. **Capital market securities** (for long-term debt or equity securities with maturities greater than 1 year):
   - Bonds;
   - Debentures;
   - Common stock;
   - Preferred stock.

1. **Primary market**:
   - Markets that involve the issue of new securities by the borrower in return for cash from investors (capital formation occurs).

2. **Secondary market**:
   - Markets that involve buyers and sellers of existing securities. Funds flow from buyer to seller. Seller becomes the new owner of the security (no capital formation occurs).

- Financial instruments are monetary contracts between parties.

- There are two major categories of financial securities:

  1. **Underlying securities**:
     - stock;
     - bonds;
     - commercial papers.

  2. **Derivative securities**:
     - forwards,
     - futures,
     - swaps;
     - options.

- A financial derivative is a contract between two (or more) parties where payment is based on (i.e., derived from) some agreed-upon underlying asset.

- Since a financial derivative can be created by means of a mutual agreement, the types of derivative products are limited only by imagination, and there is no definitive list of derivative products.

- Some common financial derivatives:
  - forwards,
  - futures,
  - swaps;
  - options.

Forward is a contract between two parties to buy or to sell an asset at a specified date future time at a price agreed upon today.

- non-standardized contract;
- traded over the counter (OTC).

Disadvantage of forwards:

- Transfer is not easy;
- Counterparty risk;
- Delivery is not easy.

- **Future** is a contract to buy or sell a standard quantity and quality of an asset or security at a specified date and price.
- Futures are similar to Forwards, but are standardized and traded on an exchange, and are valued daily. The daily value provides both parties with an accounting of their financial obligations under the terms of the Future.
- Unlike Forwards, the counterparty to the buyer or seller in a Futures contract is the clearing corporation on the appropriate exchange.
- Futures often are settled in cash or cash equivalents, rather than requiring physical delivery of the underlying asset.

Swap is a derivative contract where two parties exchange the cash flows of financial instruments:

- Interest rate swaps;
- Credit default swaps (CDS).

- **Option** is a contract which gives the buyer (the owner or holder of the option) the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified price (*strike price*) on a specified date (*maturity*).
- An Option to buy is known as a **Call**; an Option to sell is called a **Put**.
- The seller of a Call Option is obligated to sell the asset to the option buyer; the seller of a Put Option is obligated to buy the asset from option buyer.
- Options are traded on organized exchanges or the OTC.

§3 Financial derivatives

Options in our life: hotel reservation

行政间

房间36㎡ | 大床2米 | 可住：👤👤 | 楼层：5层-6层 | 包含无线

¥1291 起

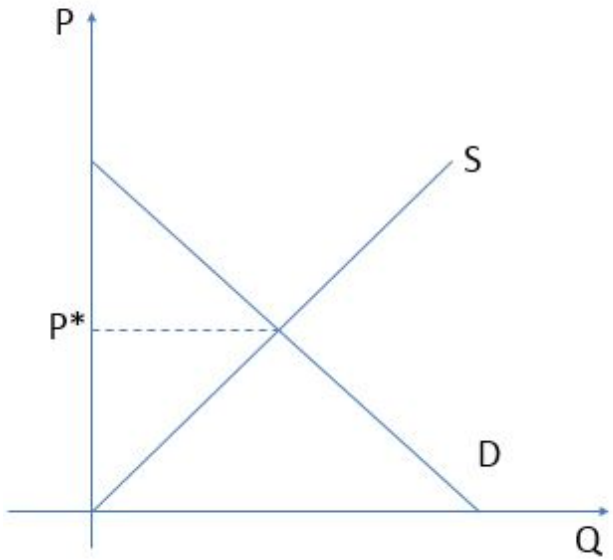
共8个产品 ▲

产品名称	供应商	早餐	取消规则	日均价		
标准价	住哪	双早	不可取消	¥1291 1400返109	担保	预订
标准价	艺龙	双早	不可取消	¥1310 1400返90	担保	预订
行政间(平台专用)(预付)	住哪	双早	不可取消	¥1341 1386返45	预付	预订
行政间(平台专用)(预付)	代理	双早	不可取消	¥1386	预付	预订
含双早	艺龙	双早	免费取消	¥1660 1750返90		预订

1. Intertemporal trade
2. Zero-sum game
3. Risky

- Hedging
- Arbitrage
- Speculation

- We use the theory of market demand and supply to price the common commodities.



- Is this available for financial derivatives?
  - No!
  - Use no-arbitrage principle to price financial derivatives.

- The term "arbitrage" is used for making risk-free profit by buying and selling financial assets in one's own account.

- Let π_t be the value of a portfolio at time $t > 0$ with $\pi_0 = 0$. An arbitrage strategy is then formally described as

$$\Pr\{\pi_t \geq 0\} = 1 \qquad \text{and} \qquad \Pr\{\pi_t > 0\} > 0.$$

- It is natural to define that, the price of an instrument is fair, if adding it to the market, does not produce arbitrage opportunities.

Assume that the same stock trades both in Chicago and in Frankfurt. The current stock price is 100 USD in Chicago and 70 EUR in Frankfurt. The EUR/USD exchange rate is 1.33 (EUR base). Neglecting transaction costs, this would imply an arbitrage opportunity as follows:

1. Buy 100 stocks in Frankfurt;
2. Immediately sell these 100 stocks in Chicago;
3. Immediately exchange the USD amount into EUR.

The resulting risk-free profit (in EUR) is

$$100 \times \left(-70 + \frac{100\ \textit{USD}}{1.33} \right) \approx 100 \times \left(-70 + 75.19 \right) = 519\ \textit{EUR} > 0.$$

- If we can construct a portfolio that has exactly the same cash flows as a given asset, this portfolio is called the *replication* of the asset.

- If the future price of the *replicating portfolio* is same as the asset, the current price of the replicating portfolio must be same as the asset; otherwise, there will be arbitrage opportunity.

- Market participants that exclusively work on exploiting arbitrage opportunities are called arbitrageurs. The presence of such arbitrageurs ensures that arbitrage opportunities disappear quickly once discovered.

- When analyzing financial markets, it is hence commonly assumed that arbitrage opportunities do not exist. This consideration is often referred as the *no-arbitrage principle*, which is the fundamental to modern pricing theory for financial markets.

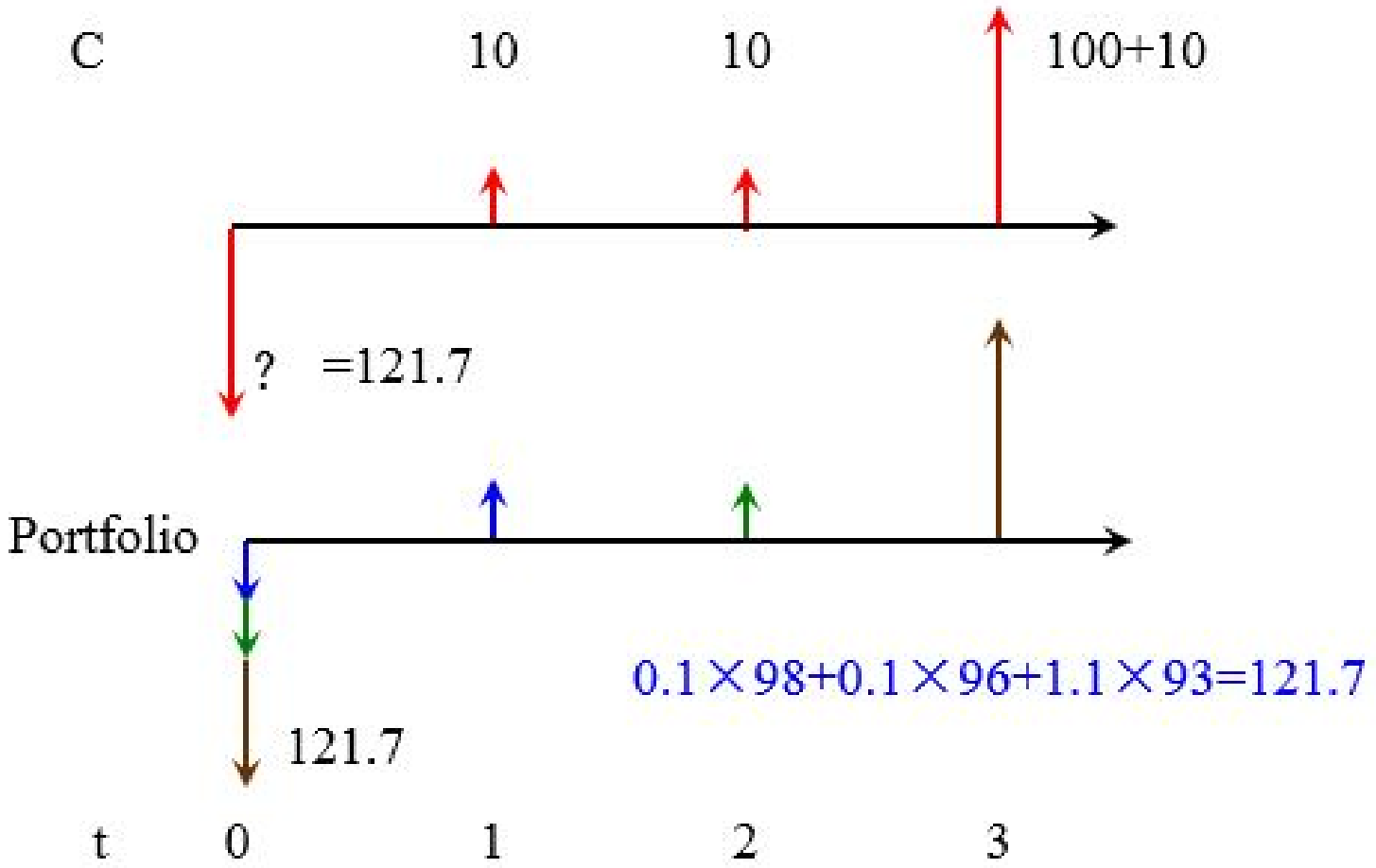
There are three (default-free) zero-coupon bonds B_1, B_2, B_3 traded in market:

- The face value of B_1, B_2 and B_3 is 100 yuan.
- The maturities of B_1, B_2 and B_3 are 1 year, 2 years, and 3 years, respectively.
- The current prices of B_1, B_2 and B_3 are 98 yuan, 96 yuan, and 93 yuan, respectively.

Assume a (default-free) company newly issues a coupon bond C, with a face value of 100 yuan, a coupon rate of 10%, with the maturity of 3 years. It pays coupon (10 yuan) once per year.

Questions:

1. What is the current fair price of C (from its buyer's point of view)?
2. Is there an arbitrage opportunity, if the current price of C is 120 yuan?
3. If there is an arbitrage opportunity, what is the arbitrage strategy?


C
10
10
100+10
? =121.7
Portfolio
0.1×98+0.1×96+1.1×93=121.7
121.7
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1
2
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- Construct a replicating portfolio consisting of B_1, B_2 and B_3:
  - Buy 0.1 B_1: after 1 year, the cash flow is $100 \times 0.1 = 10$.
  - Buy 0.1 B_2: after 2 year, the cash flow is $100 \times 0.1 = 10$.
  - Buy 1.1 B_3: after 3 year, the cash flow is $100 \times 1.1 = 110$.

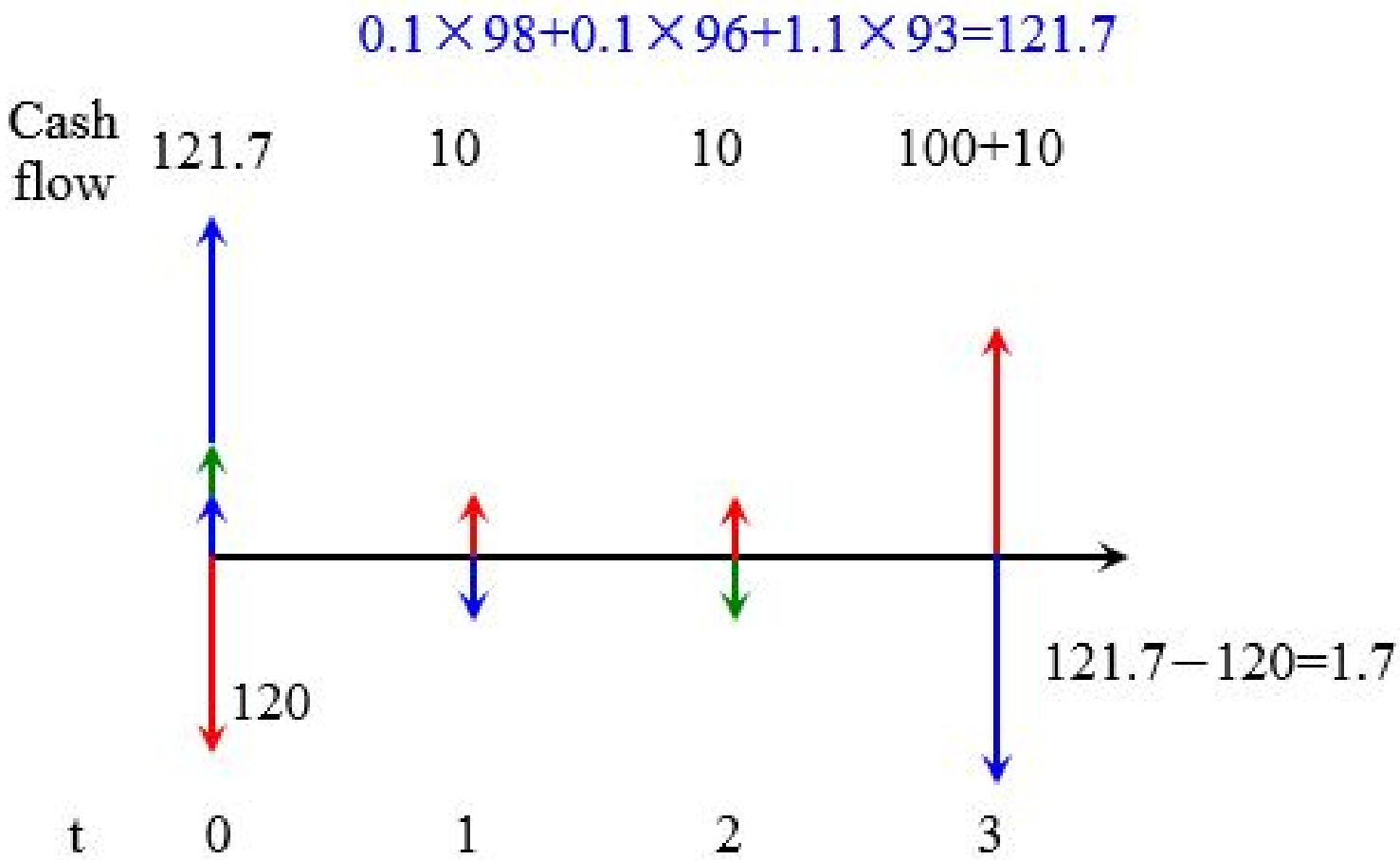
- The total current cost is

$$0.1 \times 98 + 0.1 \times 96 + 1.1 \times 93 = 121.7,$$

  which is the current fair price of C.

- The cash flows of the bond portfolio are exactly the same as C. So, we replicate coupon bond C by a portfolio of zero-coupon bonds.

- If the price of C is 120 yuan, which is less than the fair price 121.7 yuan, there is an arbitrage opportunity.
- At time 0, we can sell this bond portfolio, and buy one C:
  - Sell 0.1 B_1, we get 9.8.
  - Sell 0.1 B_2, we get 9.6.
  - Sell 1.1 B_3, we get 102.3.
  - Buy 1 C, the cost is 120.
- The risk-free profit is $121.7 - 120 = 1.7$ at time 0.


0.1×98+0.1×96+1.1×93=121.7
Cash flow
121.7
10
10
100+10
120
121.7−120=1.7
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0
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3