QUESTION BANK
Of
ABSTRACT ALGEBRAS

School of Mathematics
Yanhua Wang

Question Bank 1

上海财经大学《 Abstract Algebra 》课程考试卷闭卷

课程代码__________课程序号___________

20 ——20 学年第 学期

Name_________Student Number________Class________

No.	I	II	III	Total
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I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. There is a unique maximal ideal in a ring.

2. Let H be a subgroup of a group G, $g_1, g_2 \in G$, then $g_1H = g_2H$ is equivalent to $Hg_1^{-1} = Hg_2^{-1}$.

3. The number of left cosets is same as the number of right cosets in a group.

4. Let $|G|$=p, a prime, then G is a cyclic group.

5. Let ϕ be a ring homomorphism from a ring R to a ring S. If A is a ideal in R, then $\phi(A)$ is an ideal in S.

II. Short answer questions

1. Give the definition of a group and a ring.

2. Write down definitions of normal subgroup of a group and an ideal of a ring.

3. Find out all ideals of ring Z_6.

4. Give the cayley table of ring $(Z_3, +, \bullet)$

5. What is a maximal ideal of a ring? Give a maximal ideal of $(Z_n, +, \bullet)$.

III. Questions

1. Find out all possible homomorphism from group $\mathbb{Z}_7$ to group Z_{12}.

2. Prove that each element in U_{16} generate Z_{16}.

3. Show that subgroups of cyclic groups are cyclic.

4. What is a prime ideal of a ring? Show that : Let R be a commutative ring with identity. Then P is prime ideal in R if and only if R/P is an integral domain.

5. Let $S = R \setminus \{-1\}$, define $a \circ b = a + b + ab, \ \ \forall a, b \in \mathrm{S}$. Is $(\mathrm{S}, \circ)$ a group? Given a proof.

Question Bank 2

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I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. Identity map is an isomorphism.

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2. The intersection of two ideals is not an ideal.

3. A permutation map of a set does not need to be a bijection.

4. The character of an integral domain is prime or zero.

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5. Let Z be the set of integer numbers, then(Z, +) and (Z, -) are groups.

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II. Short answer questions.

1. Give the definition of a group and a ring.

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2. Write down the definition of normal subgroup of a group and an ideal of a ring.

3. What is a field extension and degree of an field extension? Give an example of field extension.

4. Write down the cosets of N={(1), (123),(132)} in S_3, and give the multiplication of factor group S_3/N.

5. Find out all ideals of ring Z_6.

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III. Questions

1. Show taht $Q(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in Q\}$ is a field.

2. Find out all possible homomorphism from group Z_7 to group Z_{12}.

3. Prove that there is an group isomorphism between $U(8)$ and $U(12)$.

4. A map is invertible if and only if it is both injective and surjective.

5. Show that all cyclic groups of infinite order are isomorphic to Z.

Question Bank 3

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I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. There is only a unique ideal in a ring.
2. If R is an integral domain, then R\{0}is a multiplication group.
3. There are at least two elements in a division ring.
4. Let $|G|$=p, a prime, then G is a cyclic group.
5. Let ϕ be a ring homomorphism from a ring R to a ring S. If A is a ideal in R, then $\phi(A)$ is an ideal in S.

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II. Short answer questions.

1. Give the definition of a group and a ring.

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2. Write down the definition of normal subgroup and factor group.

3. What is an ideal of a ring? Find out all ideals of ring Z_6.

4. What is the Dihedral group? Give a subgroup of a Dihedral group.

5. Explain that $Q(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in Q\}$ is a field. What is the degree of $Q(\sqrt{2})/Q$?

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III. Questions.

1. Let G be a cyclic group, then G is isomorphic to Z or Z_n .

2. What is an equivalence relation, and show that two equivalence classes of an equivalence relation are either disjoint or equal.

3. What are integral domain and division ring? Is Z an integral domain or a division ring, why?

4. Let S_3 be the permutation group and N={(1), (123), (132}

(1) Show that N is a normal subgroup of S_3, what is $[S_3 : N]$;

(2) What is the Cayley table of the factor group;

(3) What is the natural homomorphism and the kernel of the natural homomorphism.

5. Discribe the first isomorphism theorem of group and prove it.

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I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. The kernel of a group homorphism is a normal subgroup.

2. There are zero divisions in a ring.

3. A permutation map of a set does not need to be a bijection.

4. The character of an integral domain is prime or zero.
5. Let Z be the set of integer numbers, then (Z, +) and (Z, -) are groups.

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II. Short answer questions.

1. Give the definition of a group and a ring.

2. Write down the definition of normal subgroup of a group and an ideal of a ring. Find out all ideals of ring $\mathbb{Z}_6$.

3. What is Lagrange Theorem. Give a proof for Lagrange Theorem.

4. What is a maximal ideal of a ring? Give an example of maximal ideal of ring Z.

5. What is the Fundamental Theorem of Arithmetic. Explain it by an example.

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III. Questions.

1. Let R be rational number group. What is Aut(R) ?

2. Let a,b be two elements of a group G, and $aba = ba^2b$, $a^3 = 1, b^{(2n-1)} = 1$.

Then b=1.

3. What is the dihedral group D_n and its rotation subgroup R_n. Show that $D_n / \mathrm{R}_n \cong Z_2$.

4. Let R be a commutative ring with identity. Then P is prime ideal in R if and only if R/P is an integral domain.

5.Show that subgroups of cyclic groups are cyclic.

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I. Determine the following conclusions are right or wrong by T(True) or F(False).

1. There is only a unique maximal ideal in a ring.
2. There are only two cyclic group up to isomorphism.
3. The number of left cosets is same as the number of right cosets in a group.
4. Let $|G|$=p, a prime, then G is a cyclic group.
5. Every ideal of integers ring is a principle ideal.

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II. Short answer questions

1. Give the definition of a group and a ring.

2. Write down the definition of normal subgroup of a group and an ideal of a ring.

3. Find out all ideals of ring Z_6.

4. Is it true $<3,7>=<1>$ in ring Z? Explain it.

5. Prove that each element in U_{16} generate group $\mathbb{Z}_{16}$.

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III. Questions

1. What is $Aut(S_3)$?

2.. What is Lagrange Theorem. Give a proof for Lagrange Theorem.

3. A map is invertible if and only if it is both injective and surjective.

4.. Let R be a even number ring. The <4> is a maximal ideal of R, but R/<4> is not a field.

5.Show that every group is isomorphic to a group of permutation.

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Ⅰ、Determine the following conclusions are right or wrong by T(True) or F(False).

1. There are only two ideals in a divisor ring.
2. The maximal ideal of a ring is not unique.
3. The number of left cosets is same as the order of a group.
4. Let $|G|$=p, a prime, then G is a cyclic group.
5. The kernel of a ring homomorphism is an ideal.

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II. Short answer questions.

1. Write down S_3 and all subgroups of S_3.

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2. Give the definition of a group and a ring.

3.What is a factor ring .

4.Give the Caley table of ring $(Z_5, +, \bullet)$

5. What is the automorphism group of integer group $(Z, +)$.

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III. Questions

1. Prove that there is only a homomorphism

$\psi : Z_7 \to Z_{12} : a \mapsto 0$.

2.What is Lagrange Theorem. Give a proof for Lagrange Theorem.

3.There exist an infinite number of primes.

4.. Let R be a even number ring. The <4> is a maximal ideal of R, but R/<4> is not a field.

5.Show that every subgroup of a cyclic group is cyclic.

Question Bank 7

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Ⅰ、Determine the following conclusions are right or wrong by T(True) or F(False).

1. Subgroups of cyclic group are cyclic.
2. The maximal ideal of a ring is not unique.
3. A left coset of a group is a subgroup.
4. Let $|G|$=p, a prime, then G is a cyclic group.
5. The kernel of a ring homomorphism is an ideal.

Mark	

II. Short answer.

1. Let A,B,C be sets. Then

$$A(B \cap C) = (A \setminus B) \cup (A \setminus C)$$

2. Let a,b be integers and P be a prime number. If $p|ab$, then either $p|a$ or $p|b$.

3. Let a and b be nonzero integers. Then there exist integers r and s such that $\gcd(a,b)=ar+bs$, and $\gcd(a,b)$ is unique.

4.Let G be a group. Show that

$$Z(G)=\{a|a \text{ in } G, ax=xa \text{ for any } x \text{ in } G\}$$

is a subgroup of G.

4. What is Dihedral group. Write down D_5.

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III. Questions.

1. Write down the Cayley table of U(16).

2. Show that every subgroup of a cyclic group is cyclic.

3. What is a subgroup. Find out all subgroup of $(Z_{12},+)$.

4. What is a maximal ideal of a ring?

5.Let H be a subgroup of a group G. The number of left cosets of H in G is the same as the number of right cosets of H in G.

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I. Determine the following conclusions are right or wrong by T(True) or F(False).

1. Any subgroups of a cyclic group are cyclic.
2. The maximal ideal of a ring is not unique.
3. The normal subgroup is a group.
4. Let $|G|$=p, a prime, then G is a cyclic group.
5. The kernel of a group homomorphism is a ideal.

Mark	

II. Short answer questions.

1. Write down S_3 and A_3 .

2. Give the definition of a group and a ring.

3. What is a factor ring .

4. What is the field extension and degree of extension?

5.What is a generator of a group, give an example.

III. Questions.

1. Explain $Z_3 \cong \{(1),(123),(132)\}$ as group.

2. What is the group homomorphism $\phi : \mathrm{Z} \to < g >$.

3.Prove: If $\varphi : G \to H$ is a group homomorphism with kernel K , then K is normal in G. Let $\phi : G \to G/K$ be the canonical homomorphism. Then there exists a unique isomorphism $\eta : G/K \to \varphi(G), s.t. \varphi = \eta\phi$.

4.Show that: Every group is isomorphic to a group of permutations.

5. What is a prime ideal of a ring? Let R be a commutative ring with identity. Then P is prime ideal in R if and only if R/P is an integral domain

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I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. A ring has an identity.
2. Let H be a subgroup of a group G, then $|H|\,|\,|G|$.
3. The number of left cosets divides the order of the group.
4. If $|G|$=p, a prime, then G is a cyclic group.
5. The group A_4 has no group of order 4.

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II. Short answer questions.

1. Write down the definition of normal subgroup of a group and an ideal of a ring.

2. Give the definition of a group and a ring.

3. Find out all ideals of ring $\mathbb{Z}_6$.

4. Write down the definition of normal subgroup of a group and an ideal of a ring.

5. What is polynomial ring. Give an example.

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III. Questions.

1. What is Dihedral group. Show the dihedral group with order 4 that is a subgroup of symmetric group with order 4.

2. Write down the Caley table of U(16). And find out a generator of U(16).

3. Let a, b be elements of a group G such that $a^2 = b^2 = (ab)^2 = e$, $a \neq e, b \neq e$. Compute < a, b > and show < a, b > is isomorphic to a permutation group.

4. Show that every subgroup of a cyclic group is cyclic.

5. Show that every group is isomorphic to a group of permutation.

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Ⅰ、Determine the following conclusions are right or wrong by T(True) or F(False).

1. Every ring is a commutative group under addition.

2. Every field is a ring.

3. Every finite abelian group is cyclic.

4. Every polynomial with degree 2 or greater has a root in the complex numbers.

5. Every subgroup of a cyclic group is cyclic.

Mark	

II. Short answer questions.

1. Define what it means for a ring to be commutative.

2. What it means for a group to be infinite order.

3. What is a homomorphism between two groups?

4. Whta is a principle ideal.

5. Define what it means for two elements in a group to commute.

Mark	

III. Questions.

1. Let R be a commutative ring with identity. Then, every maximal ideal in R is a prime ideal.

2. Suppose G is a finite group of even order. Then, G contains an element of order 2.

3. Let R be a commutative ring with identity, and let $a \in R$. If a is not prime, then there exist elements $b,c \in R$ such that $a|bc$ but a does not divide b or c.

4. Let G be a group of order 33. Then, G is cyclic.

5. If a group G has a normal subgroup H and a cyclic subgroup K of G/H, then G has a cyclic subgroup containing H and K.

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I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. The set of integers modulo 5, denoted by $\mathbb{Z}_5$, is a field.

2. A ring with identity is a field if and only if every nonzero element in the ring has an inverse under multiplication.

3. In a group, the identity element is unique.

4. If a group has a cyclic subgroup, then the group is cyclic.

5. The order of a group is the same as the number of elements in the group.

6. The additive identity in a ring is also the multiplicative identity.

7. In a finite field, every nonzero element has a finite order.

8. In a group, if two elements have the same order, then they must be the same element.

9. The center of a group is the set of all elements that commute with every element in the group.

10. A polynomial with coefficients in a field is irreducible if and only if it has no roots in that field.

Mark	

II. Short answer questions.

1. Define what it means for a subset of a group to be a subgroup.

2.State the definition of a ring and give an example.

3. Define what it means for a group to be abelian.

4. What is an ideal in a ring?

5. Define what it means for a group to be isomorphic to another group.

Mark	

III. Questions.

1. Every cyclic group is abelian.

2. If two groups G and H are isomorphic, then they have the same order.

3. The product of two even permutations is an even permutation.

4. Let R be a ring with identity. If $a,b \in R$ and $ab=1$, then $ba=1$.

5. In a finite field, the product of all nonzero elements is 1.

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Mark	

I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. Every ring is a field.

2. An ideal of a ring contains the identity element.

3. The center of a group is always a subgroup.

4. Every field is an integral domain.

5. Every subgroup of an abelian group is normal.

Mark	

II. Short answer questions.

1. Define a group.

2.What is a ring?

3.State the multiplicative identity axiom for a field.

4.What is an ideal of a ring?

5.Define a homomorphism.

Mark	

III. Questions.

1. Prove that the group of complex numbers under multiplication is not cyclic.

2. Let R be a ring with identity. Prove that the set of units in R forms a group under multiplication.

3. Prove that any two non-identity elements in a group of order 3 generate the entire group.

4. Let I and J be ideals of a ring R. Prove that the intersection $I \cap J$ is also an ideal of R.

5. Prove that every finite field has a prime power order.

Question Bank 13

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Mark	

I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. A set with two operations is called a group.
2. Every field is also a ring.
3. A group must have an identity element.
4. Every finite group is cyclic.
5. An inverse element exists for every element in a group.

Mark	

II. Short answer questions.

1. Define a ring.

2.What is a field?

3.State the cancellation property of a ring.

4.What is the order of an element in a group?

5. Define a homomorphism of ring.

Mark	

III. Questions.

1. Prove that every subgroup of an abelian group is normal.

2. Let a and b be elements of a group G. Prove that the order of the element ab is equal to the order of the element ba.

3. Prove that the center of a non-abelian group is non-trivial.

4. Let f: G -> H be a group homomorphism. Prove that the kernel of f is a normal subgroup of G.

5. Prove that if a group G has an even number of elements, then it contains at least one element other than the identity element that is its own inve

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Mark	

I、Determine the following conclusions are right or wrong by T(True) or F(False).

1.A group must have an identity element.

2. Every finite group is cyclic.

3. An inverse element exists for every element in a group.

4. A subgroup has the same order as the original group.

5. A group is always commutative.

Mark	

II. Short answer questions.

1. What is the character of a ring.

2. What is a subgroup of a group.

3. Describe the definition of a normal subgroup.

4. What is a noncommutative ring.

Give the order of a permutation.

Mark	

III. Questions.

1.. Let CharR = p, p be a prime. Then $(a + b)p = ap + bp$, $a, b \in R$

2.What is the quotient field of Z.

3.Let a, b $\in$ Z. What is $< a, b >$?

4.Let I be an ideal of R. Define $r(I) = \{r \in R | ru = 0, \forall u \in U\}$. Prove that $r(I)$ is an ideal of R.

5.Let $\phi : Z \to Z : m \to 7m$. Prove that ϕ is a group homomorphism. Find the kernel and the image of ϕ

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I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. The ring Zhas no zero divisor.

2. Let H be a subgroup of a group G, then the order of H is a divisor of the order of G.

3. A group isomorphism is a bijection and a homomorphism of group.

4. The character of an integral domain is prime or zero.

5. Let Z be the set of integer numbers, then (Z, -) are groups.

Mark	

II. Short answer questions.

1. What is a prime ideal.

2.Give the definition of general linear group.

3. What is the center of a froup.

4. What is a factor ring.

5. Discrobe the equivalence relation.

Mark	

III. Questions.

1.In the group Z24, let H = $\langle 4\rangle$ and N = $\langle 6\rangle$. What is H + N and H ∩ N.

2. What is the Aut(Z8)? Is Aut(Z8) a cyclic group?

3.Show that $Z[i] = \{a + bi|a, b \in Z\}$ is a domain.

4.Find out all zero divisors of Z_6.

5.Find out all prime ideals of Z_{18}.

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I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. Alternating groups are simple.
2. There is no zero divisor in an integral domain.
3. Every finite integer domain is a field.
4. Let G be a group, g in G. Then $|g|\,|\,|G|$.
5. The trivial ideal is a maximal ideal.

Mark	

II. Short answer questions.

1.What are right cosets of $H = <4>$ in Z_{12}.

2. Discribe the cyclic group.

3. What is a field.

4. Give the theorem of Chinese Remainder.

5.Expalin the definition of prime ideal.

Mark	

III. Questions.

1.List the left and right cosets of subgroup $\langle 3 \rangle$ in group U(8).

2.Suppose that G is a finite group with 60 elements. What are the orders of possible subgroups of G?

3.Let G be a group and define a map $\lambda g : G \longrightarrow G$ by $\lambda g(a) = ga$. Prove that λg is a permutation of G.

4.Let I and J be ideals of ring R. Define $IJ = \{P\ i\ aibi|ai \in I, bi \in J\}$. Prove that IJ is an ideal of R

5.Let G be a finite abelian group. Prove that the product of all the elements of G equals the product of all the elements of G of order 2

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I. Determine the following conclusions are right or wrong by T(True) or F(False).

1. The character of Z_6 is 0.

2. Let I be a subring of a ring R, and J is an ideal of R, then $I \cap J$ is an ideal of I, and $I / I \cap J \cong (I+J)/J$.

3. A bijection is an injection and a surjection.

4. The character of an integral domain is prime or zero.

5. Let Z be the set of integer numbers, then (Z, -) are groups.

Mark	

II. Short answer questions.

1.What is a maximal ideal.

2.Give the definition of integral domail.

3.Discribe the zero divisor in a ring.

4.Write down S_3 and all subgroups of S_3.

5. What is a dihedral group.

Mark	

III. Questions.

1.Write down the power sets of $A = \{a, b, c, d\}$ and $B = \{\phi, \{\phi\}\}$.

2.Define the order of a set A is the number of elements in A, denote as $|A|$. How many maps from A to B if $|A| = n$ and $|B| = m$?

3.Find out all subgroups of Z8. What are cyclic subgroups of Z8?

4. Let G be a group, g in G, $|g| = mn$, and $(m, n) = 1$, then $g = ab$ where $|a| = m$, $|b| = n$, and $a, b \in G$.

5.Prove or disprove U(8) is isomorphic to Z4 as groups.

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I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. Homomorphism of group is a surjection.
2. The intersection of two ideals is not an ideal.
3. A permutation of a set does not need to be a bijection.
4. The character of an integral domain is prime.
5. The identity of a subgroups is the identity of group.

Mark	

II. Short answer questions.

1. What is a normal subgroup of a group and an ideal of a ring. Find out all ideals of ring Z_6.

2. What is a ring. Give an example of a noncommutative ring.

3. Write down the cosets of N={(1), (123), (132)} in S_3.

4. What is group isomorphism.

5. What is a divisor ring.

Mark	

III.Questions.

1.Find out all group with order 4.

2.Let R be a even ring. The <4> is a maximal ideal of R, but R/<4> is not a field.

3.Show that the characteristic of an integral domain is either prime or zero.

4.Let a, b be integers. Show that <a, b> is a principle ideal of the ring of integers.

5.Let G be a group.

(1) Show that G/Center(G) is isomorphic to Inn(G).

(2) What is $Aut(S_3)$?

Question Bank 19

上海财经大学《 Abstract Algebra 》课程考试卷闭卷

课程代码__________课程序号___________

20 ——20 学年第 学期

Name_________Student Number________Class________

No.	I	II	III	Total
Mark				

Mark	

I、Determine the following conclusions are right or wrong by T(True) or F(False).

1.The character of Z_8 is 0.

2. All automorphisms of a group is a group.

3. A permutation map of a set does not need to be a bijection.

4. The character of an integral domain is prime or zero.

5. Let Z be the set of integer numbers, then (Z, +) and (Z, -) are groups.

Mark	

II. Short answer questions.

1.Write down the dihedral group of D_5.

2.What is a normal subgroup of a group and an ideal of a ring. Find out all ideals of ring Z_6.

3.What is a ring. Give an example of a noncommutative ring.

4. Write down the cosets of N={(1), (123), (132)} in S_3, and give the multiplication of factor group S_3/N.

5. Give the Cayley table of U(8) under addition and multiplication.

Mark	

III.Questions.

1. What is a subgroup. Find out all subgroup of $(Z_{12},+)$

2. Is $\sigma = \begin{pmatrix} 1 & 2 & \cdots & n \\ 1 & n-1 & \cdots & 1 \end{pmatrix}$ an even permutation or a odd permutation?

3.A map is invertible if and only if it is both injective and surjective.

4. There exist an infinite number of primes.

5.Write down the Cayley table of U(16). And find out a generator of U(16).

Question Bank 20

上海财经大学《 Abstract Algebra 》课程考试卷闭卷

课程代码__________课程序号___________

20 ——20 学年第 学期

Name__________Student Number_________Class_________

No.	I	II	III	Total
Mark				

Mark	

I、Determine the following conclusions are right or wrong by T(True) or F(False).

1. There is only an unique maximal ideal in a ring.
2. Let H be a subgroup of a group G, $g_1, g_2 \in G$, then $g_1H = g_2H$ is equivalent to $Hg_1^{-1} = Hg_2^{-1}$.

3.The number of left cosets is same as the number of right cosets in a group.

4.Let $|G|$=p, a prime, then G is a cyclic group.

5.The order of alternating group is even number.

Mark	

II. Short answer questions.

1. Write down S_3.

2. Give the definition of a group and a ring

3. What is a factor ring .

4. Give the Cayley table of ring $(\mathbb{Z}_3, +, \bullet)$

5.What is an extension and algebraic extension.

Mark	

III. Questions.

1. If a group homomorphism is a bijection, then it is a group isomorphism.

2.The character of an integral domain is prime or zero.

3. Is it true $<3,7>=<1>$ in $(\mathbb{Z},+,\bullet)$? Explain it.

4.The intersection of two ideals is not an ideal.

5.The kernel of a group homomorphism is a normal subgroup.