

# 渐进DID模型的理论计量进展

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# Difference in Difference



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#### Basic Setup



- $t \in \{0,1\}$ : t=0 if before the treatment time; t=1 if after
- $r \in \{0,1\}$ : r=0 if in control group; r=1 if in treated group
- $\tau = t \cdot r$  is the treatment. So  $\tau=1$  only if t=1 and r=1.
- Like in experiments, only treated group after the treatment is treated.
- $Y_{\tau t}$  is the potential outcome in time t when treatment status is  $\tau$ .
- So,  $E(Y_{11} Y_{01}|r = 1)$  is the interested ATT.
- There're four observed volumes (two groups in two times):

 $\geq E(Y_{11}|r=1)$ : treatment group + after the treatment

 $\geq E(Y_{00}|r=1)$ : treatment group + before the treatment

 $\geq E(Y_{01}|r=0)$ : control group + after the treatment

 $\geq E(Y_{00}|r=0)$ : control group + before the treatment

### Identification



• We use the control group to infer the time effect of treated group. Then we can figure out the treatment effects. DID

$$= [E(Y|r = 1, t = 1) - E(Y|r = 1, t = 0)] - [E(Y|r = 0, t = 1) - E(Y|r = 0, t = 0)]$$
  
=  $E(Y_{11} - Y_{00}|r = 1) - E(Y_{01} - Y_{00}|r = 0)$   
=  $E(Y_{11} - Y_{01}|r = 1) + [E(Y_{01} - Y_{00}|r = 1) - E(Y_{01} - Y_{00}|r = 0)]$ 

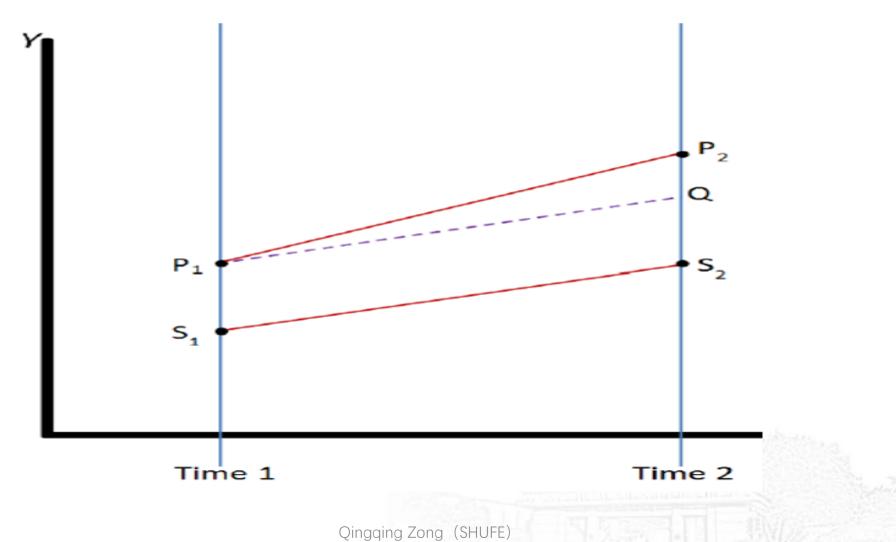
• DID is the interested ATT  $(E(Y_{11} - Y_{01}|r = 1))$  if the **Parallel Trend** Assumption holds

$$E(Y_{01} - Y_{00}|r = 1) = E(Y_{01} - Y_{00}|r = 0)$$

• The assumption means the treatment group shares the same time trend with control group, or the time trend in the absence of the intervention are the same in both groups

#### Parallel Trend Assumption





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#### Estimation



• 
$$\widehat{DID} = \left[\overline{Y}_{t_1,treated} - \overline{Y}_{t_0,treated}\right] - \left[\overline{Y}_{t_1,control} - \overline{Y}_{t_0,control}\right]$$

- Linear form:  $y = \beta_0 + \beta_t t + \beta_r r + \delta \tau + u$ ; by OLS  $\Rightarrow \hat{\delta}$
- $\hat{\delta} = \widehat{DID}$ . OLS or fixed effects can be used to estimate DID.



### Why $\delta$ is DID Estimator?

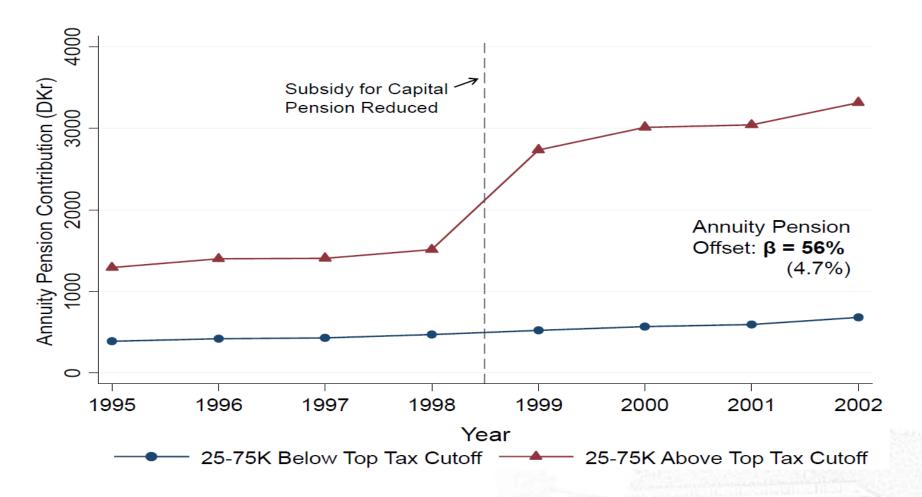


-65	Before (t=0)	After (t=1)	Difference
Treatment (r=1)	$\beta_0 + \beta_r$	$\beta_0 + \beta_r + \beta_t + \delta$	$\Delta Y_t = \beta_t + \delta$
Control (r=0)	β <sub>0</sub>	$\beta_0 + \beta_t$	$\Delta Y_c = \beta_t$
Difference	$\beta_1$	$eta_1\!\!+\!\!\delta$	$\Delta \Delta Y = \delta$
	I		

#### Test Parallel Trend



#### Impact of Capital Pension Subsidy Reduction On Annuity Pension Contributions

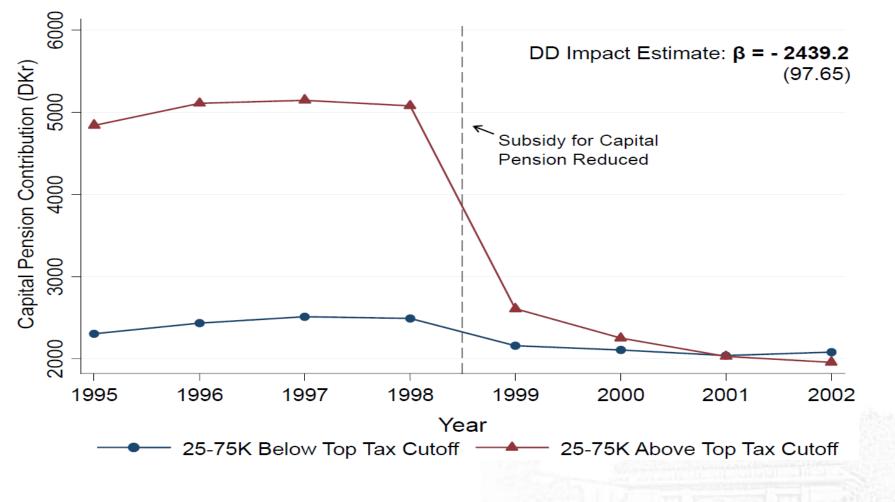


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#### Chetty et al.(2014,QJE)



Impact of Subsidy Reduction On Individual Capital Pension Contribs.





# How Much Should We Trust DID?



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### Statistical Inference of DID



- Bertrand Mariance, Esther Duflo, Sendhil Mullainathan, 2004, How Much Should We Trust Differences-in-Differences Estimates? *Quarterly Journal of Economics*, 119(1), 249-275.
- Because of **serial correlation**, DD estimation as it is commonly performed grossly under-states the standard errors around the estimated intervention effect.
- (1) First, DD estimation usually relies on fairly long time periods
- (2) Second, the most commonly used dependent variables in DD estimation are typically highly positively serially correlated
- ③ Third, the treatment variable changes itself very little within a state over time
- These three factors reinforce each other to create potentially large mismeasurement in the standard errors coming from the OLS estimation.

#### Survey of DID Paper



- Data comes from a survey of all articles in six journals between 1990 and 2000: American Economic Review; Industrial Labor Relations Review; Journal of Labor Economics; Journal of Political Economy; Journal of Public Economics; and Quarterly Journal of Economics.
- They define an article as "Difference-in-Difference" if it: (1) examines the effect of a specific interventions and (2) uses units unaffected by the intervention as a control group.
- Their survey of DD papers, which we discuss below, finds an average of 16.5 periods.

### Survey of DID Paper (cont'd)



Number of DD papers	92	
Number with more than 2 periods of data	69	
Number which collapse data into before-after	4	
Number with potential serial correlation problem	65	
Number with some serial correlation correction	5	
GLS	4	
Arbitrary variance-covariance matrix	1	
Distribution of time span for papers with more than 2 periods	Average	16.5
	Percentile	Value
	1%	3
	5%	3
	10%	4
	25%	5.75
	50%	11
	75%	21.5
	90%	36
	95%	51
	99%	83

### Over-rejection in DID Estimation



- A sample of women's wages from the Current Population Survey (CPS).
- 1979-1999, all women between the ages 25 and 50
- The sample contains nearly 900,000 observations, approximately 540,000 report strictly positive weekly earnings
- Dependent variable: log(weekly earnings)
- ① Draw a year at random from a uniform distribution between 1985 and 1995 (ensure having enough observations prior and post-intervention)
- (2) Select exactly half the states (25) at random and designate them as "affected" by the law
- ③ Estimate DD 200 times where the control variables contain education ,age , state dummies and year dummies

### Over-rejection in DID Estimation (cont'd)



	A. C	PS DATA		
			Rejecti	on rate
Data	$\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3$	Modifications	No effect	2% effect
1) CPS micro, log			.675	.855
wage			(.027)	(.020)
2) CPS micro, log		Cluster at state-	.44	.74
wage		year level	(.029)	(.025)
3) CPS agg, log	.509, .440, .332		.435	.72
wage			(.029)	(.026)
4) CPS agg, log	.509, .440, .332	Sampling	.49	.663
wage		w/replacement	(.025)	(.024)
5) CPS agg, log	.509, .440, .332	Serially	.05	.988
wage		uncorrelated laws	(.011)	(.006)
6) CPS agg,	.470, .418, .367		.46	.88
employment			(.025)	(.016)
7) CPS agg, hours	.151, .114, .063		.265	.280
worked			(.022)	(.022)
8) CPS agg, changes	046, .032, .002		0	.978
in log wage				(.007)

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### Over-rejection in DID Estimation (cont'd)



- The stylized exercise above focused on data with 50 states
- and 21 time periods.
- Many DD papers use fewer states and several DD papers use fewer time periods.
- They examined how the rejection rate varies with these two important parameters
- ① Varying the number of states does not change the extent of the over-rejection
- 2 Over-rejection falls as the time span gets shorter, but it does so at a rather slow rate

### Over-rejection in DID Estimation (cont'd)



A. CPS DATA         1) CPS aggregate       50       21       .49       .663         2) CPS aggregate       20       21       .39       .54         2) CPS aggregate       20       21       .39       .54         3) CPS aggregate       10       21       .443       .510         4) CPS aggregate       6       21       .383       .433         5) CPS aggregate       6       21       .383       .433         6) CPS aggregate       50       11       .20       .638         6) CPS aggregate       50       7       .15       .635         7) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       3       .048       .363         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       (.011)       (.024)         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       .603       .603       .603         9) CPS aggregate       50       2       .055       .28         (.011)       (.0220)       .6028)       .6028)       .6028) </th <th></th> <th>•</th> <th>RYING N AND</th> <th></th> <th></th>		•	RYING N AND		
A. CPS DATA         1) CPS aggregate       50       21       .49       .663         2) CPS aggregate       20       21       .39       .54         2) CPS aggregate       20       21       .39       .54         3) CPS aggregate       10       21       .443       .510         4) CPS aggregate       6       21       .383       .433         5) CPS aggregate       6       21       .383       .433         6) CPS aggregate       50       11       .20       .638         6) CPS aggregate       50       7       .15       .635         7) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       3       .048       .363         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       (.011)       (.024)         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       .603       .603       .603         9) CPS aggregate       50       2       .055       .28         (.011)       (.0220)       .6028)       .6028)       .6028) </th <th></th> <th></th> <th></th> <th>Rejecti</th> <th>on rate</th>				Rejecti	on rate
1) CPS aggregate       50       21       .49       .663         2) CPS aggregate       20       21       .39       .54         3) CPS aggregate       10       21       .443       .510         3) CPS aggregate       10       21       .443       .510         4) CPS aggregate       6       21       .383       .433         5) CPS aggregate       50       11       .20       .638         5) CPS aggregate       50       7       .15       .635         6) CPS aggregate       50       7       .15       .635         6) CPS aggregate       50       5       .078       .5         7) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       2       .055       .28         9) CPS aggregate       50       21       .35       .638         (011)       (.024)       .0011       (.024)         9) CPS aggregate       50       21       .35       .28         10       AR(1), $\rho = .8$ 50       21       .35       .538         11) AR(1), $\rho = .8$ 6       21       .3975       .505         12)	Data	N	т	No effect	2% effect
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2) CPS aggregate 20 21 .39 .54 .(024) (025) 3) CPS aggregate 10 21 .443 .510 .(025) (025) 4) CPS aggregate 6 21 .383 .443 .(025) (025) 5) CPS aggregate 50 11 .20 .638 .(020) (024) 6) CPS aggregate 50 7 .15 .635 .(017) (024) 7) CPS aggregate 50 5 .078 .5 .(013) (025) 8) CPS aggregate 50 3 .048 .363 .(011) (.024) 9) CPS aggregate 50 2 .055 .28 .(011) (.022) B. MONTE CARLO SIMULATIONS WITH SAMPLING FROM AR(1) DISTRIBUTION 10) AR(1), $\rho = .8$ 50 21 .35 .638 .(028) (029) 11) AR(1), $\rho = .8$ 10 21 .35 .638 .(028) (029) 12) AR(1), $\rho = .8$ 50 .11 .35 .505 .(028) .(028) .(028) .(029) 13) AR(1), $\rho = .8$ 50 .50 .11 .35 .588 .(029) 14) AR(1), $\rho = .8$ 50 .50 .50 .208 .(020) .209 .(021) .200 .(021) .200 .(021) .200 .(022) .200 .(022) .200 .(023) .200 .(023) .200 .(023) .200 .(024) .200 .(025) .200 .(026) .200 .(027) .200 .200 .(027) .200 .200 .(027) .200	1) CPS aggregate	50	21	.49	.663
(.024)       (.025)         3) CPS aggregate       10       21       .443       .510         (.025)       (.025)       (.025)       (.025)         4) CPS aggregate       6       21       .383       .443         5) CPS aggregate       50       11       .20       .638         5) CPS aggregate       50       7       .15       .635         6) CPS aggregate       50       7       .15       .635         7) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       2       .055       .28         9) CPS aggregate       50       2       .055       .28         9) CPS aggregate       50       21       .35       .638         10) AR(1), $\rho = .8$ 50       21       .35       .638         11) AR(1), $\rho = .8$ 50       21       .35       .538         12) AR(1), $\rho = .8$ 6       21       .393       .5				(.025)	(.024)
3) CPS aggregate       10       21       .443       .510         4) CPS aggregate       6       21       .383       .433         5) CPS aggregate       6       21       .383       .433         6) CPS aggregate       50       11       .20       .638         6) CPS aggregate       50       7       .15       .638         7) CPS aggregate       50       7       .15       .638         7) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       3       .048       .363         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       (.024)       (.024)         9) CPS aggregate       50       3       .048       .363         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       .022)       .022)       .022)         B. MONTE CARLO SIMULATIONS WITH SAMPLING FROM AR(1) DISTRIBUTION       10) AR(1), $\rho = .8$ 50       21       .35       .538         11) AR(1), $\rho = .8$ 10       21       .3275       .505       .505         12) AR(1), $\rho = .8$ 6 <td><ol><li>CPS aggregate</li></ol></td> <td>20</td> <td>21</td> <td>.39</td> <td>.54</td>	<ol><li>CPS aggregate</li></ol>	20	21	.39	.54
(1) $2 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + $				(.024)	(.025)
4) CPS aggregate 6 21 $(.383)$ $(.433)$ (.025) $(.025)5) CPS aggregate 50 11 20 (.024)6) CPS aggregate 50 7 .15 .635(.017)$ $(.024)7) CPS aggregate 50 5 .078 .5(.017)$ $(.024)7) CPS aggregate 50 3 .048 .3638) CPS aggregate 50 2 .011 (.025)8) CPS aggregate 50 2 .011 (.024)9) CPS aggregate 50 2 .055 .28(.011)$ $(.024)9) CPS aggregate 50 2 .055 .28(.011)$ $(.022)B. MONTE CARLO SIMULATIONS WITH SAMPLING FROM AR(1) DISTRIBUTION10) AR(1), \rho = .8 50 21 .35 .63811) AR(1), \rho = .8 10 21 .35 .63812) AR(1), \rho = .8 6 21 .35 .53513) AR(1), \rho = .8 6 21 .355 .53814) AR(1), \rho = .8 6 21 .3933 .515) AR(1), \rho = .8 50 .11 .335 .58816) AR(1), \rho = .8 50 .11 .335 .58817) AR(1), \rho = .8 50 .3 .099 .43518) AR(1), \rho = .8 50 .11 .325 .582519) AR(1), \rho = .8 50 .11 .335 .58810) AR(1), \rho = .8 50 .11 .325 .582511) AR(1), \rho = .8 50 .11 .325 .582512) AR(1), \rho = .8 50 .175 .552513) AR(1), \rho = .8 50 .3 .099 .435514) AR(1), \rho = .8 50 .3 .099 .435515) AR(1), \rho = .8 50 .3 .099 .435516) AR(1), \rho = .8 50 .3 .099 .435517) AR(1), \rho = .8 50 .50 .4975 .8557$	<ol><li>CPS aggregate</li></ol>	10	21	.443	.510
1       0.025       (.025)       (.025)         5) CPS aggregate       50       11       .20       .638         6) CPS aggregate       50       7       .15       .635         7) CPS aggregate       50       7       .15       .635         7) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       3       .048       .363         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       .0011       (.024)         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       .20       .28       .011       .024)         9) CPS aggregate       50       21       .35       .638         10       AR(1), $\rho = .8$ 50       21       .35       .538         11) AR(1), $\rho = .8$ 10       21       .3975       .505         12) AR(1), $\rho = .8$ 6       21       .393       .5         (.028)       (.029)       .0028)       .0029)         14) AR(1), $\rho = .8$ 50       5       .175       .5525         15) AR(1), $\rho = .8$				(.025)	(.025)
5) CPS aggregate       50       11       .20       .638         6) CPS aggregate       50       7       .15       .638         6) CPS aggregate       50       7       .15       .638         7) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       3       .048       .363         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       .0013       (.025)         8) CPS aggregate       50       3       .048       .363         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       .0025       .28       .011       .024)         9) CPS aggregate       50       2       .0655       .28       .28       .011       .022)         B. MONTE CARLO SIMULATIONS WITH SAMPLING FROM AR(1) DISTRIBUTION       .021       .35       .538       .638       .028)       .0293       .028)       .028)       .0293       .028)       .0293       .0293       .0293       .0293       .0293       .595       .505       .175       .5525       .2525       .022)       .0293       .0293       .525       .	<ol><li>CPS aggregate</li></ol>	6	21	.383	.433
(.020)       (.024)         (.017)       (.024)         (.017)       (.024)         (.017)       (.024)         (.017)       (.024)         (.017)       (.024)         (.013)       (.025)         (.013)       (.025)         (.011)       (.024)         (.013)       (.025)         (.011)       (.024)         (.013)       (.025)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.024)         (.011)       (.022)         (.028)       (.029)         (.028)       (.029)         (.028)       (.029)         (.028)       (.029)         (.021)       .028)       (.022)         (.022)       (.028)         (.022)       (.029) </td <td></td> <td></td> <td></td> <td>(.025)</td> <td>(.025)</td>				(.025)	(.025)
6) CPS aggregate       50       7       .15       .635         7) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       3       .048       .363         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       .013)       (.025)         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)       .055       .28       .011)       (.024)         9) CPS aggregate       50       2       .055       .28       .011)       (.024)         9) CPS aggregate       50       21       .35       .638       .28       .011)       (.022)         B. MONTE CARLO SIMULATIONS WITH SAMPLING FROM AR(1) DISTRIBUTION       .028)       (.022)       .028)       .029         11) AR(1), $\rho = .8$ 20       21       .35       .538       .538         12) AR(1), $\rho = .8$ 10       21       .3975       .505         13) AR(1), $\rho = .8$ 50       11       .335       .588         (.027)       (.028)       .0291       .0291       .0291         14) AR(1), $\rho = .8$ 50       3	5) CPS aggregate	50	11	.20	.638
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				(.020)	(.024)
7) CPS aggregate       50       5       .078       .5         8) CPS aggregate       50       3       .048       .363         9) CPS aggregate       50       2       .055       .28         (.011)       (.024)         9) CPS aggregate       50       2       .055       .28         (.011)       (.022)         B. MONTE CARLO SIMULATIONS WITH SAMPLING FROM AR(1) DISTRIBUTION       10) AR(1), $\rho = .8$ 50       21       .35       .638         10) AR(1), $\rho = .8$ 50       21       .35       .638         11) AR(1), $\rho = .8$ 20       21       .35       .538         12) AR(1), $\rho = .8$ 10       21       .3975       .505         13) AR(1), $\rho = .8$ 6       21       .393       .5         (.028)       (.029)       (.028)       (.029)         14) AR(1), $\rho = .8$ 50       11       .335       .588         15) AR(1), $\rho = .8$ 50       5       .175       .5525         (.022)       (.029)       .09       .4355         16) AR(1), $\rho = .8$ 50       3       .09       .4355         17) AR(1), $\rho = .8$ 50       50	6) CPS aggregate	50	7	.15	.635
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				(.017)	(.024)
8) CPS aggregate 50 3 .048 .363 9) CPS aggregate 50 2 .055 .28 (.011) (.024) B. MONTE CARLO SIMULATIONS WITH SAMPLING FROM AR(1) DISTRIBUTION 10) AR(1), $\rho = .8$ 50 21 .35 .638 (.028) (.028) 11) AR(1), $\rho = .8$ 20 21 .35 .538 (.028) (.029) 12) AR(1), $\rho = .8$ 10 21 .35 .538 (.028) (.029) 13) AR(1), $\rho = .8$ 6 21 .393 .5 (.028) (.029) 14) AR(1), $\rho = .8$ 50 11 .335 .585 (.027) (.028) 15) AR(1), $\rho = .8$ 50 5 .175 .5525 (.027) (.028) 16) AR(1), $\rho = .8$ 50 3 .09 .4355 (.017) (.029) 17) AR(1), $\rho = .8$ 50 50 .4975 .855	7) CPS aggregate	50	5	.078	.5
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				(.013)	(.025)
9) CPS aggregate 50 2 .055 .28 (.011) (.022) B. MONTE CARLO SIMULATIONS WITH SAMPLING FROM AR(1) DISTRIBUTION 10) AR(1), $\rho = .8$ 50 21 .35 .638 (.028) (.028) 11) AR(1), $\rho = .8$ 20 21 .35 .538 (.028) (.029) 12) AR(1), $\rho = .8$ 10 21 .3975 .055 13) AR(1), $\rho = .8$ 6 21 .393 .5 (.028) (.029) 14) AR(1), $\rho = .8$ 50 11 .335 .638 (.028) (.029) 14) AR(1), $\rho = .8$ 50 .11 .335 .0588 (.027) (.028) 15) AR(1), $\rho = .8$ 50 .5 .175 .5525 (.022) (.029) 16) AR(1), $\rho = .8$ 50 .3 .09 .4355 17) AR(1), $\rho = .8$ 50 .50 .4975 .655	8) CPS aggregate	50	3	.048	.363
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				(.011)	(.024)
B. MONTE CARLO SIMULATIONS WITH SAMPLING FROM AR(1) DISTRIBUTION10) AR(1), $\rho = .8$ 5021.35.63811) AR(1), $\rho = .8$ 2021.35.53812) AR(1), $\rho = .8$ 1021.3975.50513) AR(1), $\rho = .8$ 621.393.514) AR(1), $\rho = .8$ 5011.335.58815) AR(1), $\rho = .8$ 505.175.52516) AR(1), $\rho = .8$ 503.09.43517) AR(1), $\rho = .8$ 5050.4975.855	9) CPS aggregate	50	2	.055	.28
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	10) AR(1), $\rho = .8$	50	21	.35	.638
11) $AR(1), \rho = .8$ 2021.35.53812) $AR(1), \rho = .8$ 1021.3975.50513) $AR(1), \rho = .8$ 621.393.514) $AR(1), \rho = .8$ 5011.335.58815) $AR(1), \rho = .8$ 505.175.5028)16) $AR(1), \rho = .8$ 503.09.43517) $AR(1), \rho = .8$ 5050.4975.855					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	11) AR(1), $\rho = .8$	20	21		
12) $AR(1), \rho = .8$ 1021.3975.50513) $AR(1), \rho = .8$ 621.393.514) $AR(1), \rho = .8$ 5011.335.58815) $AR(1), \rho = .8$ 505.175.552516) $AR(1), \rho = .8$ 503.09.43517) $AR(1), \rho = .8$ 5050.09.435					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	12) AR(1), $\rho = .8$	10	21		
13) $AR(1), \rho = .8$ 621.393.514) $AR(1), \rho = .8$ 5011.335.58815) $AR(1), \rho = .8$ 505.175.552516) $AR(1), \rho = .8$ 503.09.43517) $AR(1), \rho = .8$ 5050.4975.855					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	13) AR(1), $\rho = .8$	6	21		
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$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	14) AR(1), $\rho = .8$	50	11		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	15) AR(1), $\rho = .8$	50	5		
16) $AR(1), \rho = .8$ 503.09.43517) $AR(1), \rho = .8$ 5050.4975.855					
17) AR(1), $\rho = .8$ 50 50 (.017) (.029) .855	<b>16</b> ) AR(1), $\rho = .8$	50	3		
17) $AR(1), \rho = .8$ 50 50 .4975 .855					
	17) AR(1), $\rho = .8$	50	50		
(.029) (.020)				(.029)	(.020)

#### TABLE III VARYING N AND T

#### Solutions



- Parametric Methods: misspecification??
- Block Bootstrap: complicated!
- Ignoring Time Series Information

#### Parametric Methods



			Rejecti	ion rate
Data	Technique	Estimated $\hat{\rho}_1$	No effect	2% Effect
	A. CPS I	DATA		
) CPS aggregate	OLS		.49	.663
			(.025)	(.024)
) CPS aggregate	Standard AR(1)	.381	.24	.66
	correction		(.021)	(.024)
CPS aggregate	AR(1) correction		.18	.363
	imposing $\rho = .8$		(.019)	(.024)
B. 0	THER DATA GENER	RATING PROCI	ESSES	
) AR(1), $\rho = .8$	OLS		.373	.765
			(.028)	(.024)
) AR(1), $\rho = .8$	Standard AR(1)	.622	.205	.715
	correction		(.023)	(.026)
$AR(1), \rho = .8$	AR(1) correction		.06	.323
	imposing $\rho = .8$		(.023)	(.027)
) AR(2), $\rho_1 = .55$	Standard AR(1)	.444	.305	.625
$\rho_2 = .35$	correction		(.027)	(.028)
AR(1) + white	Standard AR(1)	.301	.385	.4
noise, ρ = .95, noise/signal = .13	correction		(.028)	(.028)

#### Block Bootstrap



- For each placebo intervention , compute the absolute *t*-statistic
- Construct a bootstrap sample by drawing with replacement 50 matrices (y\_s,v\_s)
- Run OLS on this sample, obtain an estimate beta\_r\_hat, t\_r = abs(beta\_r\_hat - beta\_hat)/se(beta\_r\_hat)
- The difference between this distribution and the sampling distribution of t becomes small as N goes to infinity, even in the presence of arbitrary autocorrelation

### Block Bootstrap(cont'd)



			Rejecti	ion rate
Data	Technique	N	No effect	2% effect
	A. CPS DA	TA		
1) CPS aggregate	OLS	50	.43	.735
			(.025)	(.022)
2) CPS aggregate	Block bootstrap	50	.065	.26
			(.013)	(.022)
3) CPS aggregate	OLS	20	.385	.595
			(.022)	(.025)
4) CPS aggregate	Block bootstrap	20	.13	.19
			(.017)	(.020)
5) CPS aggregate	OLS	10	.385	.48
			(.024)	(.024)
6) CPS aggregate	Block bootstrap	10	.225	.25
			(.021)	(.022)
7) CPS aggregate	OLS	6	.48	.435
			(.025)	(.025)
8) CPS aggregate	Block bootstrap	6	.435	.375
			(.022)	(.025)
	B. AR(1) DISTR	IBUTION	1	
9) AR(1), $\rho = .8$	OLS	50	.44	.70
			(.035)	(.032)
10) AR(1), $\rho = .8$	Block bootstrap	50	.05	.25
			(.015)	(.031)

#### TABLE V BLOCK BOOTSTRAP

### Ignoring Time Series Information



#### IGNORING TIME SERIES DATA

			Rejecti	ion rate
Data	Technique	N	No effect	2% effect
	A. CPS DATA			
1) CPS agg	OLS	50	.49	.663
			(.025)	(.024)
2) CPS agg	Simple aggregation	50	.053	.163
			(.011)	(.018)
3) CPS agg	Residual aggregation	50	.058	.173
0.000		20	(.011)	(.019)
4) CPS agg, staggered laws	Residual aggregation	50	.048	.363
E) CDS area	OLS	20	(.011)	(.024)
5) CPS agg	OLS	20	.39	.54
6) CPS agg	Simple aggregation	20	(.025) .050	(.025)
6) CFS agg	Simple aggregation	20	(.011)	(.014)
7) CPS agg	Residual aggregation	20	.06	.183
() 015 agg	incolution appregation	20	(.011)	(.019)
8) CPS agg, staggered laws	Residual aggregation	20	.048	.130
e, ere ugg, suggered hans	incondum upprogrammin		(.011)	(.017)
9) CPS agg	OLS	10	.443	.51
-/			(.025)	(.025)
10) CPS agg	Simple aggregation	10	.053	.065
			(.011)	(.012)
11) CPS agg	Residual aggregation	10	.093	.178
			(.014)	(.019)
<ol><li>CPS agg, staggered laws</li></ol>	Residual aggregation	10	.088	.128
			(.014)	(.017)
13) CPS agg	OLS	6	.383	.433
			(.024)	(.024)
14) CPS agg	Simple aggregation	6	.068	.07
			(.013)	(.013)
15) CPS agg	Residual aggregation	6	.11	.123
			(.016)	(.016)
16) CPS agg, staggered laws	Residual aggregation	6	.09	.138
B.	AR(1) DISTRIBUTION		(.014)	(.017)
17) AR(1), $\rho = .8$	Simple aggregation	50	.050	.243
	Pro Cop control		(.013)	(.025)
18) AR(1), $\rho = .8$	Residual aggregation	50	.045	.235
	-000		(.012)	(.024)
19) AR(1), $\rho = .8$ , staggered laws	Residual aggregation	50	.075	.355
	00 0		(.015)	(.028)



# Staggered DID



Qingqing Zong (SHUFE)

#### Basic Setup



- The canonical difference-in-differences (DID) model contains two time periods, "pre" and "post", and two groups, "treatment" and "control".
- Most DID applications, however, exploit variation across groups of units that receive treatment at different times.
- Staggered DID adopts a two-way fixed effects specification:

$$y_{it} = \alpha_i + \lambda_t + \beta D_{it} + u_{it}$$

## Use of DID in Finance and Accounting: 2000-2019



	(1)	(2)	(3)
	DiD	Staggered	Staggered DiD / DiD
	DID	$\operatorname{DiD}$	(%)
Journal of Finance	54	29	53.70%
Journal of Financial Economics	162	79	48.77%
Review of Financial Studies	139	66	47.48%
Review of Finance	28	12	42.86%
Journal of Financial and Quantitative Analysis	56	32	57.14%
Finance	<b>439</b>	<b>218</b>	49.66%
Journal of Accounting Research	52	21	40.38%
Journal of Accounting and Economics	63	34	53.97%
The Accounting Review	108	52	48.15%
Review of Accounting Studies	46	24	52.17%
Contemporary Accounting Research	43	17	39.53%
Accounting	<b>312</b>	148	47.44%
Finance and Accounting	751	366	48.74%

#### Recent Advances



- Goodman-Bacon, A.2021, Difference-in-Differences with Variation in Treatment Timing. *Journal of Econometrics*
- Baker, Andrew C., David F. Larcker, and Charles C.Y. Wang. "How Much Should We Trust Staggered Difference-In-Differences Estimates?" 2021,Working Paper

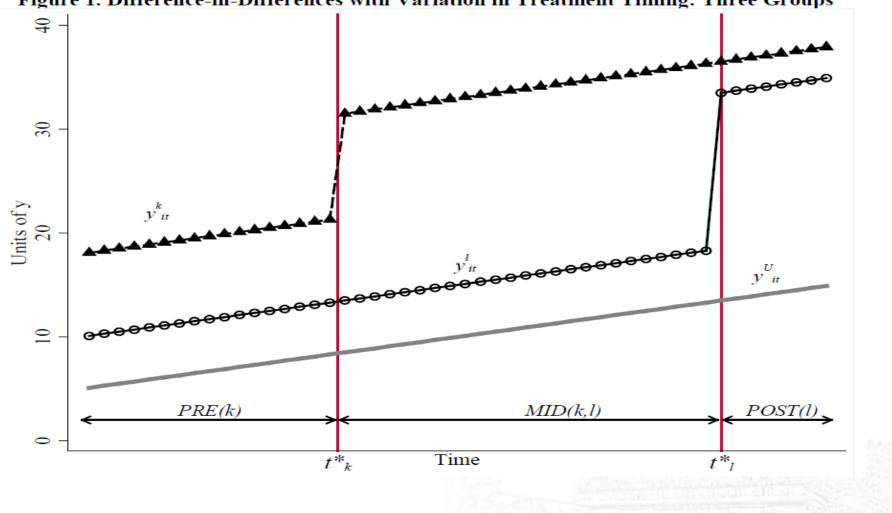
#### Main Conclusions

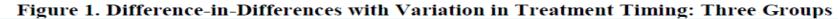


- Recent advances in econometric theory show that such designs are likely to be biased in the presence of treatment effect heterogeneity
- Goodman-Bacon(2021) derives an expression for this general DID estimator according to the *DD Decomposition Theorem*, and shows that it is a weighted average of all possible two-group/ two-period DID estimators in the data
- Baker et al.(2021) apply recently proposed methods to a set of prior published results and find that the reported effects in prior research become indistinguishable from zero in many cases

#### Three Groups

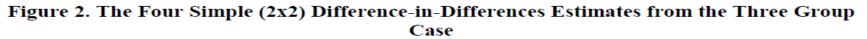


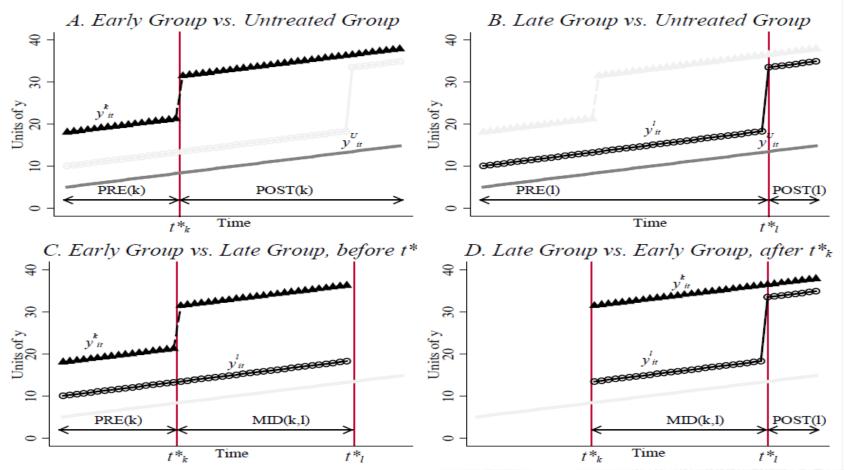




#### Decomposition: Graph







# Decomposition



$$\widehat{\boldsymbol{\beta}}^{DD} = \sum_{k \neq U} s_{kU} \,\widehat{\boldsymbol{\beta}}_{kU}^{2x2} + \sum_{k \neq U} \sum_{\ell > k} s_{k\ell} \left[ \mu_{k\ell} \,\widehat{\boldsymbol{\beta}}_{k\ell}^{2x2,k} + (1 - \mu_{k\ell}) \,\widehat{\boldsymbol{\beta}}_{k\ell}^{2x2,\ell} \right]$$

$$\widehat{\boldsymbol{\beta}}_{kU}^{2x2} \equiv \left(\overline{\boldsymbol{y}}_{k}^{POST(k)} - \overline{\boldsymbol{y}}_{k}^{PRE(k)}\right) - \left(\overline{\boldsymbol{y}}_{U}^{POST(j)} - \overline{\boldsymbol{y}}_{U}^{PRE(j)}\right)$$

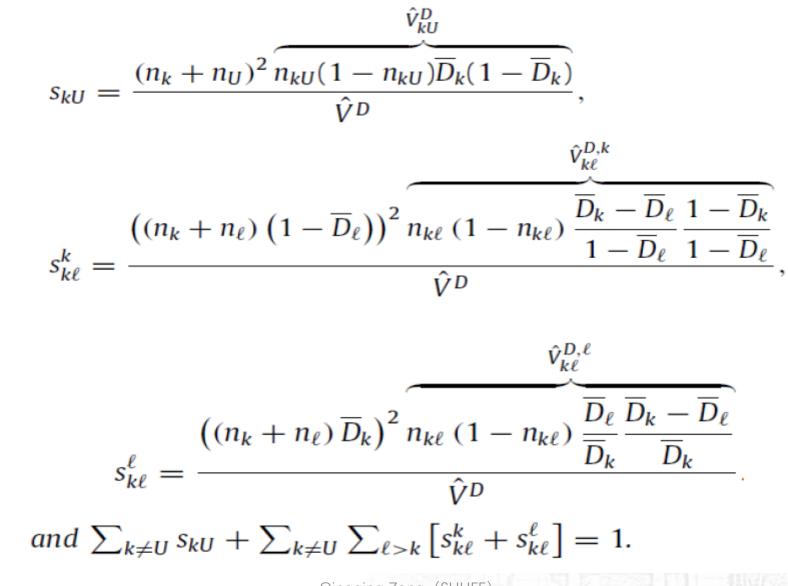
$$\widehat{\boldsymbol{\beta}}_{k\ell}^{2x2,k} \equiv \left( \overline{\boldsymbol{y}}_{k}^{MID(k,\ell)} - \overline{\boldsymbol{y}}_{k}^{PRE(k)} \right) - \left( \overline{\boldsymbol{y}}_{\ell}^{MID(k,\ell)} - \overline{\boldsymbol{y}}_{\ell}^{PRE(k)} \right)$$

$$\widehat{\boldsymbol{\beta}}_{k\ell}^{2x2,\ell} \equiv \left( \overline{\boldsymbol{y}}_{\ell}^{POST(\ell)} - \overline{\boldsymbol{y}}_{\ell}^{MID(k,\ell)} \right) - \left( \overline{\boldsymbol{y}}_{k}^{POST(\ell)} - \overline{\boldsymbol{y}}_{k}^{MID(k,\ell)} \right)$$

Qingqing Zong (SHUFE)

#### Weights



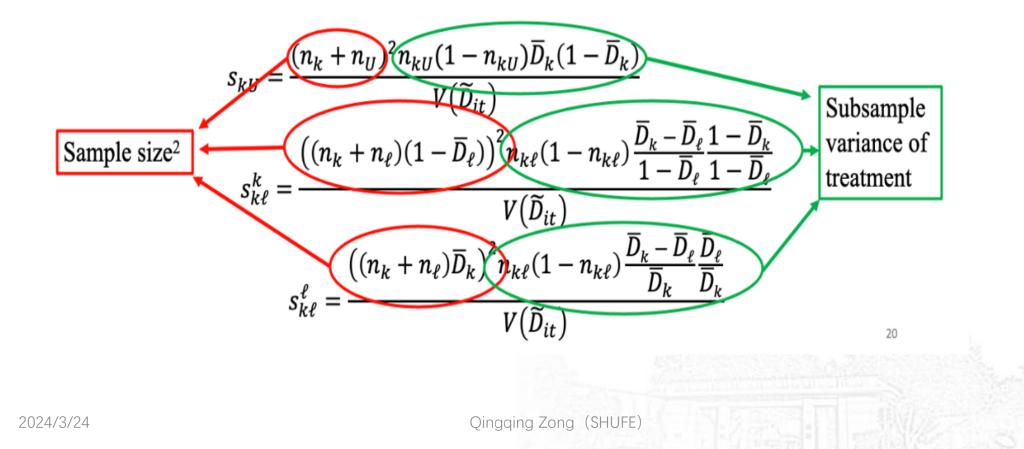


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### Weights (cont'd)



• The weights come both from group sizes and the treatment variance in each pair





$$ATT_k(W) \equiv E[Y_{it}^1 - Y_{it}^0 | k, t \in W]$$

Is the estimated parameter ATT?

$$\Delta Y_k^h(W_1, W_0) \equiv E[Y_{it}^h | k, W_1] - E[Y_{it}^h | k, W_0], \quad h = 0, 1$$

$$\beta_{kU}^{2x2} = ATT_k(POST(k)) + \Delta Y_k^0(POST(k), PRE(k)) - \Delta Y_U^0(POST(k), PRE(k))$$
(11a)

$$\beta_{k\ell}^{2x2,k} = ATT_k \left( MID(k,\ell) \right) + \Delta Y_k^0 \left( MID(k,\ell), PRE(k) \right) - \Delta Y_\ell^0 \left( MID(k,\ell), PRE(k) \right) (11b)$$

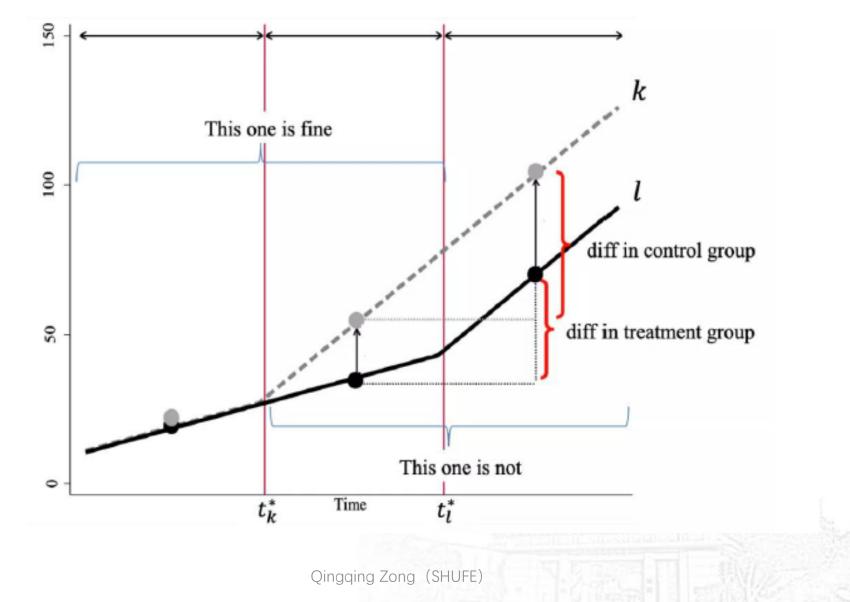
$$\beta_{k\ell}^{2x2,\ell} = ATT_{\ell}(POST(\ell)) + \Delta Y_{\ell}^{0}(POST(\ell), MID(k,\ell)) - \Delta Y_{k}^{0}(POST(\ell), MID(k,\ell))$$

$$-\left[ATT_{k}(POST(\ell)) - ATT_{k}(MID(k,\ell))\right]$$
(11c)

$$\mathcal{A}^{\ell} = ATT_{\ell}(POST(\ell)) + \Delta Y_{\ell}^{0}(POST(\ell), MID(k, \ell)) - \Delta Y_{k}^{0}(POST(\ell), MID(k, \ell))$$

#### The Bias





#### The Meaning of Beta



 $plim \ \hat{\beta}^{DD} = VWATT + VWCT - \Delta ATT$ 

(1) Variance-weighted ATT

$$VWATT \equiv \sum_{k \neq U} \sigma_{kU} ATT_k (POST(k)) + \sum_{k \neq U} \sum_{\ell > k} \left[ \sigma_{k\ell}^k ATT_k (MID(k,\ell)) + \sigma_{k\ell}^\ell ATT_l (POST(k)) \right]$$

(2) Variance-weighted common trends

$$\begin{aligned} VWCT &\equiv \sum_{k \neq U} \sigma_{kU} \left[ \Delta Y_k^0 \left( POST(k), PRE(k) \right) - \Delta Y_U^0 \left( POST(k), PRE(k) \right) \right] \\ &+ \sum_{k \neq U} \sum_{\ell > k} \left[ \sigma_{k\ell}^k \left\{ \Delta Y_k^0 \left( MID(k,l), PRE(k) \right) - \Delta Y_l^0 \left( MID(k,l), PRE(k) \right) \right\} \\ &+ \sigma_{k\ell}^l \left\{ \Delta Y_l^0 \left( POST(l), MID(k,l) \right) - \Delta Y_k^0 \left( POST(l), MID(k,l) \right) \right\} \right] \approx \sum_k \Delta Y_k^0 [w_k^T - w_k^C] \end{aligned}$$

(3) Weighted sum of the change in treatment effects

$$\Delta ATT \equiv \sum_{k \neq U} \sum_{\ell > k} \sigma_{k\ell}^{\ell} \left[ ATT_k \left( POST(\ell) \right) - ATT_k \left( MID(k, \ell) \right) \right]$$

总结



- 多时点DID的估计本质上是多个传统2×2 DID估计的加权平
   均,其权重是子样本规模、处理组与控制组相对规模、以及
   子样本方差的函数:子样本规模越大,相互比较的两个组子样
- 本规模越近,处于中期(如两次政策实施时点之间)的处理组会被赋予更高的权重
- 已经被处理的观测还能作为"控制组",即便他们本身并不 是控制组
- 多时点DID方法极易产生研究偏误,尤其是在政策渐进实施 过程中处理效应变化的情况下(Heterogeneous treatment effect)
- 只有满足平行趋势及零时变处理效应,系数才能被解读为平均处理效应(政策带来的因果效应)!!!
- Stata 命令: help bacondecomp

#### No-fault divorce reforms and female suicide



#### Table 1. The No-Fault Divorce Rollout: Treatment Times, Group Sizes, and Treatment Shares

No-Fault Divorce	Number of	Share of	Treatment Share
Year $(t_k^*)$	States	States $(n_k)$	$(\overline{D}_k)$
Non-Reform States	5	0.10	
Pre-1964 Reform States	8	0.16	
1969	2	0.04	0.85
1970	2	0.04	0.82
1971	7	0.14	0.79
1972	3	0.06	0.76
1973	10	0.20	0.73
1974	3	0.06	0.70
1975	2	0.04	0.67
1976	1	0.02	0.64
1977	3	0.06	0.61
1980	1	0.02	0.52
1984	1	0.02	0.39
1985	1	0.02	0.36

Notes: The table lists the dates of no-fault divorce reforms from Stevenson and Wolfers (2006), the number and share of states that adopt in each year, and the share of periods each treatment timing group spends treated in the estimation sample from 1964-1996.

### bacondecomp



							*	Variables	T 4 X
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. help baconde	comp							year	
								_nfd	
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								asmrs	
. help baconde	comp							pcinc	
. xtreg asmrs	nost naina a	emrh cases i	vear f	e robust				asmrh	
· Acteg domito	post perme a	Smirn Cubeb r	.ycar, r	C LODUDO				cases	
Fixed-effects	(within) reg	ression		Number	of obs =	1,617		weight	
Group variable	: stfips			Number	of groups =	49		сорор	
R-sq:				Obs per	group:		=		
within =	0.3655				min =	33			
between =	0.0032				avg =	33.0			
overall =	0.1375				max =	33			
				F(36,48	) =	19.40			
corr(u_i, Xb)	= -0.1421			Prob >	F =	0.0000		* III.	*
								Properties	<b>4</b> ×
		(Std. 1	Err. adju	usted for	49 clusters	in stfips)			
		Robust						Variables	
asmrs	Coef.	Std. Err.	t	P>[t]	[95% Conf.	Interval]		Name	
								Label	
post	-2.515964	2.283101	-1.10	0.276	-7.106446	2.074519		Туре	
pcinc	0011182	.0003614	-3.09	0.003	0018448	0003916		Format	
	1.184598	.5765446	2.05	0.045	.0253772	2.343819		Value label	
asmrh cases	-178.5179	137.9739	-1.29	0.202	-455.933	98.89717			

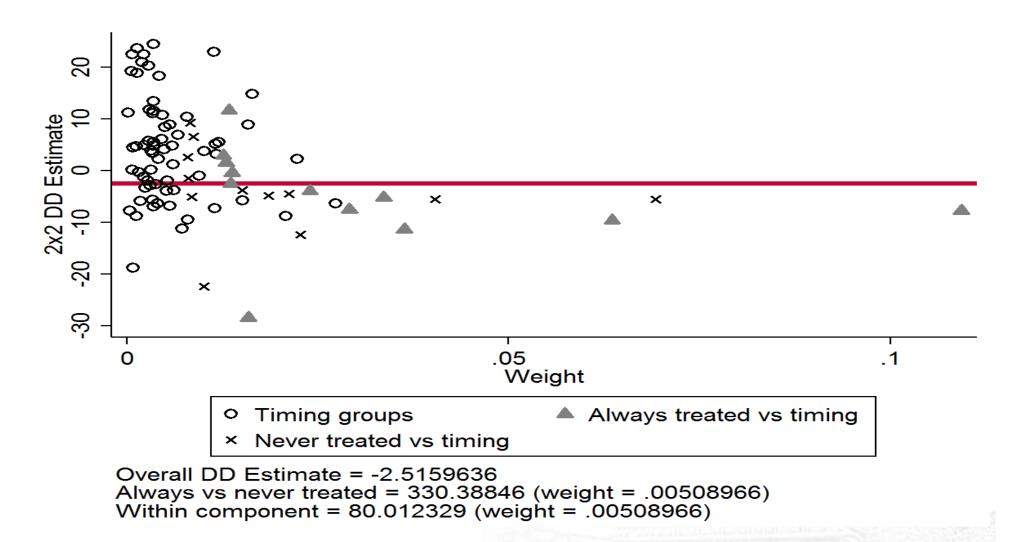
#### bacondecomp (cont'd)



asmrs	Coef	. Std. Err.	z	P> z	[95% Conf.	Interval]
post	-2.51596	54 2.283101	-1.10	0.270	-6.99076	1.958833
on Decompos	sition					
on Decompos	ition					
on Decompos	ition	Beta	TotalW	eight		
	sition	Beta 2.602167327		_		
Timir			.37766	06651		
Timir Always_	1g_groups	2.602167327	.37766	06651 86972		
Timir Always Never	ng_groups _v_timing	2.602167327 -7.02043576	.37766 .37830 .23892	06651 86972 29013		

#### bacondecomp (cont'd)

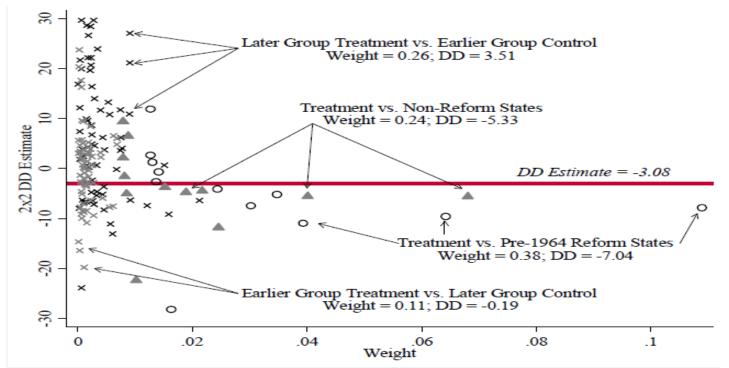




#### DID decomposition



Figure 6. Difference-in-Differences Decomposition for Unilateral Divorce and Female Suicide



Notes: Notes: The figure plots each 2x2 DD components from the decomposition theorem against their weight for the unilateral divorce analysis. The open circles are terms in which one timing group acts as the treatment group and the pre-1964 reform states act as the control group. The closed triangles are terms in which one timing group acts as the treatment group and the non-reform states act as the control group. The x's are the timing-only terms. The figure notes the average DD estimate and total weight on each type of comparison. The two-way fixed effects estimate, -3.08, equals the average of the y-axis values weighted by their x-axis value.



# Thanks!

