实物期权及企业投资决策

第七讲 二叉树期权定价模型

- A one-step Binomial Model
  - Assume no arbitrage opportunities exit.
  - Refer to figure 1.
    - Set up a portfolio of stock and option in such a way that there is no uncertainty about the value of portfolio at the end of the 3 months.
    - How to calculate the value of the call option?
  - A Generalization
    - Refer to figure 2.

$$f = e^{-rT}[pf_u + (1-P)f_d]$$

    - We can interpret P as the probability of an up movement in the stock price, and 1-P is the probability of a down movement, then equation states the value of option today is its expected future value discounted at the risk-free rate.

- How about the stock price is when the probability of up movement is assumed to be P.
- P is called the risk-neutral probability measure. Under this measure, all individuals are indifference to risk.
- Note: the resulting option price are correct not just in a risk-neutral world , but in the real world as well.

- Two –Step Binomial Trees
  - Refer to figure 3, figure 4.
  - A generalization (the case of European option)
    - Refer to figure 5
    - The option price is always equal to its expected payoff in a risk-neutral world, discounted at the risk-free interest rate.

- The case of American option
  - Refer to figure 6, figure 7.
  - The value of the final nodes is the same as for the European option.
  - At earlier nodes the value of the option is the greater of
    - 1. the value given by equation $f = e^{-rT}[pf_u + (1-P)f_d]$
    - 2. the payoff from early exercise.
- Delta: the delta of a stock potion is the ratio of change in the price of the stock option to the change in the price of the underlying stock. It is same as the Δ aforementioned.
  - Delta changes over time.
  - In order to maintain a risk-less hedge using an option and underlying stock, the holdings in the stock must be adjusted at the end of each step.

- Matching volatility with u and d.
  - Choose the parameters u and d match the volatility of the stock price.
  - Refer to figure 8.
  - When we move from the real world to risk-neutral world the expected return changes, but its volatility remains the same.
  - Girsanovs theorem:
    - When we move from a world with one set of risk preference to a world with another set of preferences, the expected growth rates in variables change, but their volatilities remain the same.

- In practice, the life of the option is typically divided into 30 or more time steps.

$$
\begin{aligned}
P &= \frac{a-d}{u-d} \\
u &= e^{\sigma\sqrt{\Delta t}} \\
d &= e^{-\sigma\sqrt{\Delta t}} \\
a &= e^{r\Delta t}
\end{aligned}
$$

- Tree of stock prices.
  - Refer to figure 9, figure 10.
  - At the time zero, the stock price S_0 is known. At the time $i\Delta t$, i+1 stock price are considered. These are,

$S_0 u^i d^{i-j}$, $i = 0, 1, \cdots, i$

Note the relationship $u = 1/d$

- Working Backward Through The Tree.
  - Option are evaluated by starting at the end of the tree and working backward.
  - The value at each node at time $T - n\Delta t$ can be calculated as the expected value at time $T-(n-1)\Delta t$ discounted for a time period Δt at rate r.
  - If the option is American, it is necessary to check at each node to see whether early exercise is preferable to holding the option for a further time period Δt.
- Expressing the Approach Algebraically.

Assuming no early exercise,

$$f_{i,j} = e^{-r\Delta t}[Pf_{i+1,j+1} + (1-P)f_{i+1,j}$$
$$0 \le \le N,\ 0 \le j \le i$$

When early exercise is taken into account,

$$f_{i,j} = \max\{X - S_0 u^j d^{i-j}, e^{-r\Delta t}[Pf_{i+1,j+1} + (1-P)f_{i+1,j}\}$$
$$0 \le \le N,\ 0 \le j \le i$$

Figure 1

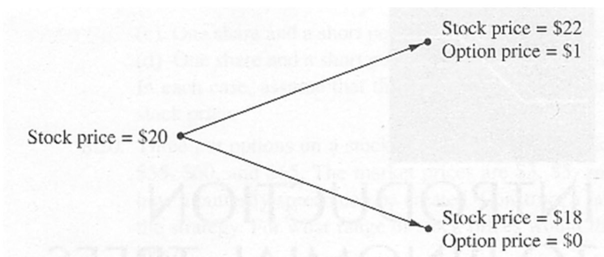


Figure 9.1 Stock price movements in numerical example.

Figure 2

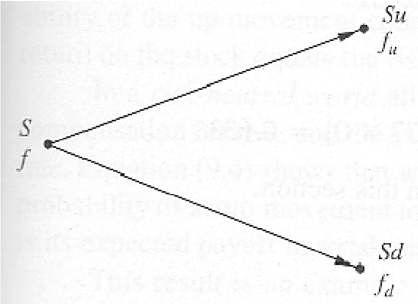
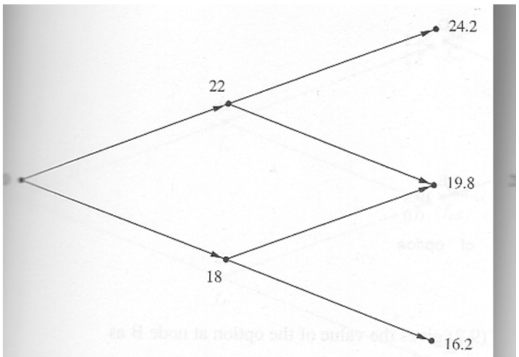


Figure 9.2 Stock and option prices in a general one-step tree.

Figure 3



gure 9.3 Stock prices in a two-step tree.

Figure 4

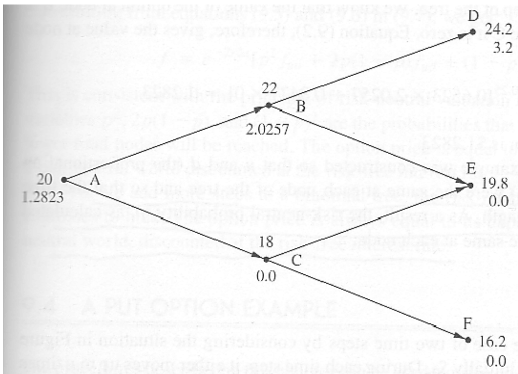


Figure 9.4 Stock option prices in a two step tree. The upper number at each node is the stock price; the lower number is the option price.

Figure 5

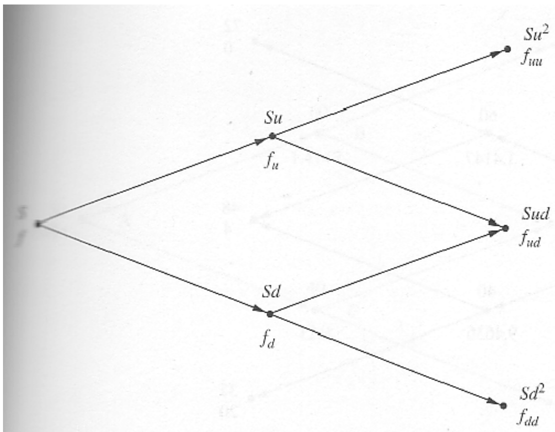


Figure 9.6 Stock and option prices in a general two-step tree.

Figure 6

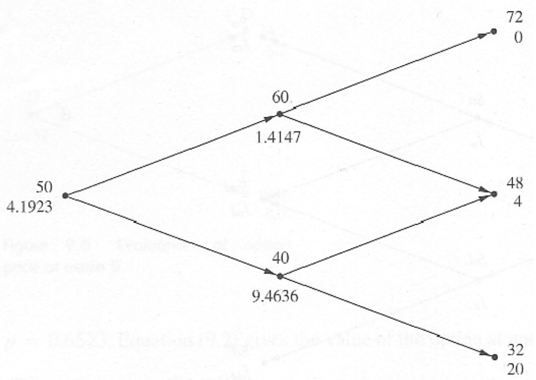


Figure 9.7 Use of two-step tree to value a European put option. At each node the upper number is the stock price; the lower number is the option price.

Figure 7

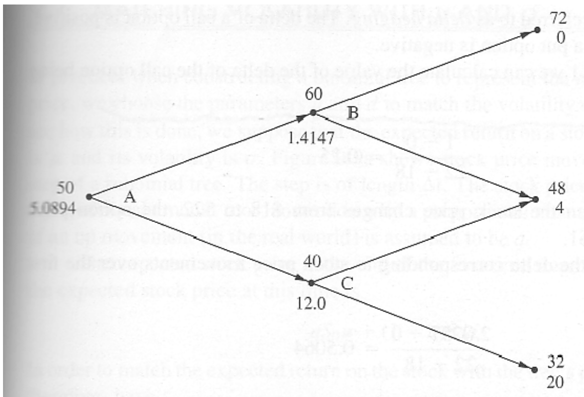


Figure 9.8 Use of two-step tree to value an American put option. At each node the upper number is the stock price; the lower number is the option price.

Figure 8

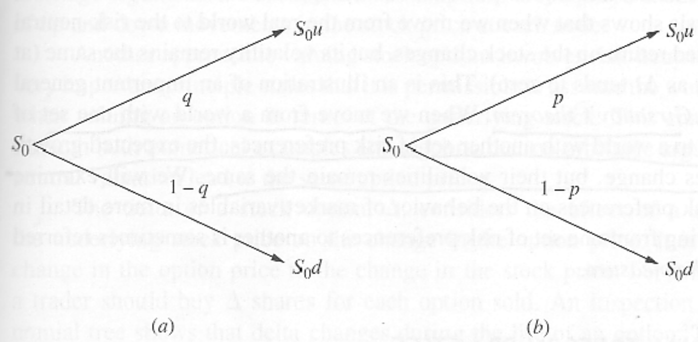


Figure 9.9 Change in stock price in time Δt in (a) the real world and (b) the risk-neutral world.

Figure 9

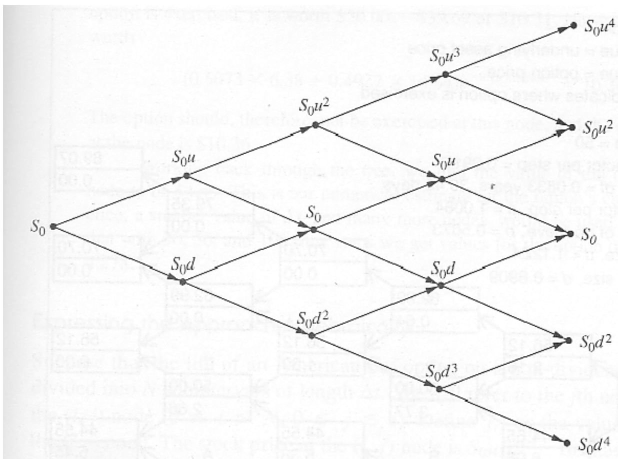


Figure 16.2 Tree used to value a stock option

Figure 10

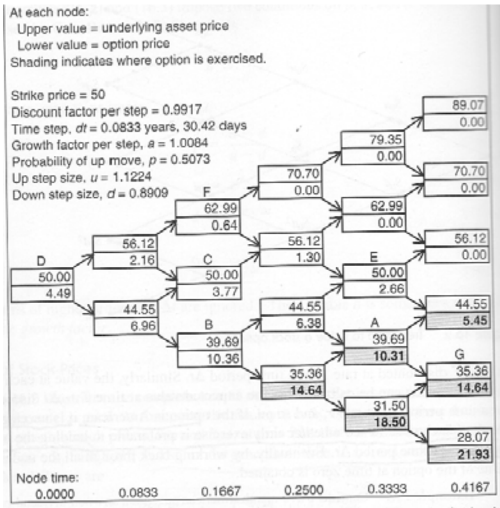


Figure 16.3 Binomial tree from DerivaGem for American put on non-dividend-paying stock (Example 16.1).