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Donald J. Smith

ABSTRACT

One of the niceties of fixed-income analysis is that one can do a great deal simply with the mathematics of the price-yield relationship. Even without a formal asset pricing model and the inevitable assumptions about behavior and information, one can assess how much a bond price will change for a given shift in its yield to maturity. In this article, I use this mathematical relationship to derive closed-form formulas for bond duration and convexity statistics, not just for coupon dates but also for days between coupons. There are two motivations for doing this: (1) to obtain formulas easily programmed onto a computer spreadsheet or a financial calculator, and (2) to demonstrate the source of the statistics for bond market professionals who rely largely on pre-programmed technology in their work. Comparable closed-form formulas have appeared in the academic literature.[1] The results derived here, while not new theoretical findings, are presented in a unified and, hopefully, simplified manner.

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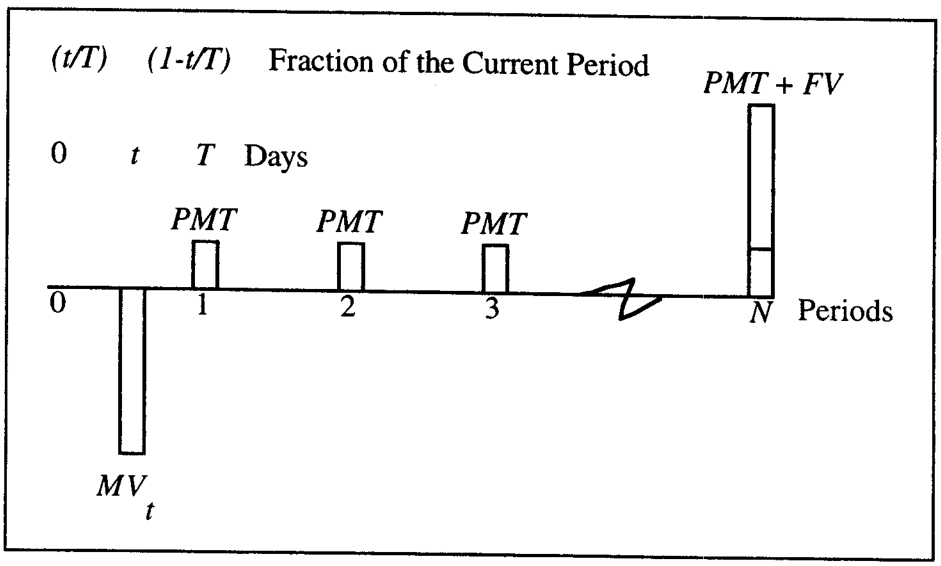
I. THE GENERAL BOND PRICE-YIELD RELATIONSHIP

Following notation familiar to users of financial calculators, let FV be the face value, or par value, of a bond payable in full at maturity, PMT the fixed coupon payment per period (so that PMT/FV is the coupon rate per period), N the number of remaining coupons, and i the yield per period. Also, let t be the number of days since the last coupon payment and T be the number of days in the coupon period. Therefore, t/T is the fraction of the period that has elapsed and $1 - t/T$ is the fraction remaining. Finally, let MV_t be the market value of the bond, that is, the total purchase or sale price if the bond were to be traded for settlement on date t, inclusive of accrued interest. This notation is summarized in Exhibit 1.

The yield to maturity on the bond is the internal rate of return of the cash flows, the solution for i in the following expression:

$$MV_t = \frac{PMT}{(1+i)^{1-t/T}} + \frac{PMT}{(1+i)^{2-t/T}} + \ldots + \frac{PMT + FV}{(1+i)^{N-t/T}} \tag{1}$$

Exhibit 1
Notation



The market value is simply the sum of the present values of the future cash flows discounted back to date t at rate i. The forthcoming coupon payment is discounted for the remaining fraction of the period, $1 - t/T$, the subsequent payment for that remaining fraction plus another full period, and so forth. Notice that the market value here is the total cash ("dirty") price paid to buy the bond at date t, inclusive of accrued interest. As such, it is not the price typically quoted in the financial press and on traders' screens. That price, the so-called "clean" price, subtracts off accrued interest (the "dirt") from the last coupon date. However, market practice is to calculate accrued interest on a straight-line basis (e.g., $t/T \times PMT$) and thereby neglect the time value of money.[2]

The numerator and denominator in Equation 1 can be multiplied by $(1 + i)^{t/T}$ to simplify:

$$MV_t = \left[\frac{PMT}{(1+i)^1} + \frac{PMT}{(1+i)^2} + \ldots + \frac{PMT + FV}{(1+i)^N}\right] \times (1+i)^{t/T} \qquad (2)$$

The sum of the present values in brackets in Equation 2 can be defined as PV_0, the price of the bond if N full periods remained until maturity. So, the general relationship between the market value of a bond and its yield to maturity can be written as:

$$MV_t = PV_0 \times (1+i)^{t/T} \qquad (3)$$

Note that PV_0 is not the historical price on date 0; rather, it is the hypothetical price that would have prevailed on the last coupon date if the yield to maturity had been i. The closed-form solution for PV_0 shown in Equation 4 facilitates calculation and is used in the derivations to follow:

$$PV_0 = \left[\frac{PMT}{i} \times \left(1 - \frac{1}{(1+i)^N}\right)\right] + \frac{FV}{(1+i)^N} \qquad (4)$$

II. BOND DURATION

The change in the market value of the bond (dMV_t) given a change in its yield to maturity (di), assuming no change in the timing and amount of the scheduled coupon and principal payments (i.e., given PMT, FV, N, and t/T), can be approximated by the first two terms of a general Taylor's series expansion:

$$dMV_t \approx \left[\frac{\partial MV_t}{\partial i} \times (di)\right] + \left[\frac{1}{2} \times \frac{\partial^2 MV_t}{\partial i^2} \times (di)^2\right] \quad (5)$$

The first term in brackets captures the bond's *duration,* the second term the bond's *convexity.* Note that these relate the change in yield to the total market value including accrued interest, and not just to the quoted book value.

The first partial derivative of the market value to a change in the yield can be obtained from general bond price-yield relationship developed in Equation 3, that $MV_t = PV_0 \times (1 + i)^{t/T}$:

$$\frac{\partial MV_t}{\partial i} = \left(\frac{\partial PV_0}{\partial i} \times (1 + i)^{t/T}\right) + \left(t/T \times PV_0 \times (1 + i)^{t/T - 1}\right) \quad (6)$$

Multiplying both sides of Equation 6 by $(1 + i)$, dividing by MV_t, and simplifying gives us:

$$\frac{\partial MV_t}{\partial i} \times \left(\frac{1 + i}{MV_t}\right) = \left(\frac{\partial PV_0}{\partial i} \times \frac{(1 + i)}{PV_0}\right) + t/T \quad (7)$$

Now *define* the duration statistics corresponding to date 0 and date t to be the following:

$$DUR_0 \equiv -\left(\frac{\partial PV_0}{\partial i} \times \frac{(1 + i)}{PV_0}\right) \quad (8)$$

and

$$DUR_t \equiv -\left(\frac{\partial MV_t}{\partial i} \times \frac{(1 + i)}{MV_t}\right) \quad (9)$$

Substituting Equations 8 and 9 into Equation 7 provides a result for duration between coupon dates:

$$DUR_t = DUR_0 - t/T \tag{10}$$

The duration on any date t in the period is the duration that would have prevailed on date 0 at the beginning of the period if the yield to maturity at that time had been i, less the fraction of the period that has elapsed. Note that the duration statistic would decay linearly as time passes over the coupon period if the yield were constant. In that case, DUR_0 would be unchanged and the t/T term would increase smoothly from zero to one as days go by. On the coupon date, however, there will be a discrete jump as the number of periods remaining drops by one, changing DUR_0.

A closed-form equation for DUR_0 is derived in Appendix A and shown below in Equation 11. The duration statistic is a function of only three variables: the yield per period i, the coupon rate per period c (where $c = PMT/FV$), and the number of periods until maturity N.

$$DUR_0 = \frac{1 + i}{i} - \frac{1 + i + [N \times (c - i)]}{[c \times ((1 + i)^N - 1)] + i} \tag{11}$$

Two properties of duration are apparent in this expression. The duration of a zero-coupon bond (i.e., $c = 0$) is equal to the time to maturity, N. Also, the duration of a perpetuity is simply the first term, $(1 + i)/i$, since the second term approaches zero as N approaches infinity.

III. BOND CONVEXITY

The convexity of the bond can be derived in a similar manner. The second partial derivative of the market value to a change in the yield is:

$$\begin{aligned}\frac{\partial^2 MV_t}{\partial i^2} = &\left[2 \times \left(\frac{\partial PV_0}{\partial i} \times t/T \times (1 + i)^{t/T - 1}\right)\right] \\ &+ \left(\frac{\partial^2 PV_0}{\partial i^2} \times (1 + i)^{t/T}\right) \\ &+ \left(t/T \times (t/T - 1) \times PV_0 \times (1 + i)^{t/T - 2}\right)\end{aligned} \tag{12}$$

Now *define* the convexity statistics for date 0 and date t:

$$CONV_0 \equiv \left(\frac{\partial^2 PV_0}{\partial i^2} \times \frac{1}{PV_0} \right) \quad (13)$$

and

$$CONV_t \equiv \left(\frac{\partial^2 MV_t}{\partial i^2} \times \frac{1}{MV_t} \right) \quad (14)$$

Next substitute Equations 13 and 14 into Equation 12 and rearrange terms using the basic result that $MV_t = PV_0 \times (1 + i)^{t/T}$ and the definition of duration in Equation 8 to obtain:

$$CONV_t = CONV_0 - \left(\frac{t/T}{(1 + i)^2} \times \left[\left(2 \times DUR_0\right) + \left(1 - t/T\right) \right] \right) \quad (15)$$

The convexity statistic between coupon dates is a function of the yield to maturity i, the fraction of the period elapsed t/T, and the duration and convexity of the bond calculated for the beginning of the period based on the yield on date t. The key point is that closed-form, easily programmed formulas for DUR_0 and $CONV_0$ are available, the former in Equation 11, the latter in Equation 16 as derived in Appendix B.

$$CONV_0 = \frac{\left[2 \times c \times (1 + i)^2 \times \left((1 + i)^N - \frac{1 + i + (i \times N)}{1 + i} \right) \right] + \left[N \times (N + 1) \times i^2 \times (i - c) \right]}{i^2 \times (1 + i)^2 \times \left\{ \left[c \times \left((1 + i)^N - 1 \right) \right] + i \right\}} \quad (16)$$

In the special case of a zero-coupon bond ($c = 0$), $CONV_0$ reduces to just $N \times (N + 1)$ divided by $(1 + i)^2$.

The connection between the duration and convexity statistics and changes in the periodic yield and market value can be summarized by substituting Equations 9 and 14 into Equation 5:

$$dMV_t \approx \left[-\left(\frac{DUR_t}{1 + i} \times MV_t \right) \times (di) \right] + \left[\frac{1}{2} \times \left(CONV_t \times MV_t \right) \times (di)^2 \right] \tag{17}$$

DUR_t as defined here is equivalent to what is generally known as *Macaulay duration* after Frederick Macaulay who first used it as a measure of interest rate risk. DUR_t divided by one plus the yield per period is known as *modified duration.* Modified duration multiplied by the market value of the security is the *money* (or *dollar*) *duration.* Similarly, the convexity statistic times the market value can be called *money convexity* (or *dollar convexity*).

IV. AN EXAMPLE USING DURATION AND CONVEXITY STATISTICS

As a numerical example of how these formulas can be used, suppose that the 8% U.S. Treasury bond maturing on November 15, 2021, happened to yield exactly 6.00% for settlement on February 29, 1996. That date was the 106th day into the 182-day coupon period between November 15, 1995, and May 15, 1996. To get the market value of the bond, substitute $FV = 100$ (the assumed par value), $PMT = 4$ (since the coupons are paid semiannually), $i = .03$ (since the quoted yield of 6.00 percent is an annualized percentage rate, i.e., the yield per semiannual period times two periods in the year), and $N = 52$ (the number of semiannual periods from the last coupon date until maturity) into Equation 4 to get $PV_0 = 126.166240$. The market value of the bond on February 29, 1996, would have been 128.357067, calculated as $126.166240 \times (1.03)^{106/182}$.

To solve for the bond's duration, substitute $i = .03$, $N = 52$, and $c = .04$ into Equation 11:

$$DUR_0 = \frac{1.03}{.03} - \frac{1 + .03 + \left[52 \times (.04 - .03)\right]}{\left[.04 \times \left((1.03)^{52} - 1\right)\right] + .03} = 25.528 \tag{18}$$

Since the fraction of the period that had elapsed was 106/182, the Macaulay duration would have been 24.946:

$$DUR_t = 25.528 - (106/182) = 24.946 \tag{19}$$

This duration statistic is in terms of semiannual periods since it used the interest and coupon rates per period and number of periods to maturity as inputs. The annualized version of the statistic is shown in the next section.

To obtain the convexity statistic, substitute the same variables into Equation 16:

$$CONV_0 = \frac{\left[2 \times .04 \times (1.03)^2 \times \left((1.03)^{52} - \frac{1.03 + (.03 \times 52)}{1.03} \right)\right] + \left[52 \times 53 \times (.03)^2 \times (.03 - .04)\right]}{(.03)^2 \times (1.03)^2 \times \left\{\left[.04 \times \left((1.03)^{52} - 1\right)\right] + .03\right\}} = 931.161 \tag{20}$$

The convexity, as of February 29, 1996, is found to be 902.903 via Equation 15:

$$\begin{aligned} CONV_t &= 931.161 - \left(\frac{106/182}{(1.03)^2} \times \left[(2 \times 25.528) + (1 - 106/182)\right] \right) \\ &= 902.903 \end{aligned} \tag{21}$$

Now suppose that an investor owning this Treasury bond wanted to estimate the loss if the yield to maturity were to have jumped suddenly to 6.50%. That implies a change of 25 basis points in the yield per semiannual period. Substitute $di = .0025$, $DUR_t = 24.946$, $MV_t = \$128.357067$, and $CONV_t = 902.903$ into Equation 15 to get a change in market value in the amount of \$7.409664:

$$\begin{aligned} dMV_t \approx & \left[-\left(\frac{24.946}{1.03} \times \$128.357067 \right) \times (.0025) \right] \\ & + \left[\frac{1}{2} \times (902.903 \times \$128.357067) \times (.0025)^2 \right] \\ = & -\$7.771833 + \$0.362169 \\ = & -\$7.409664 \end{aligned} \tag{22}$$

The actual market value would have been \$120.934619 at a new yield of 6.50 percent, a change of –\$7.422448. Notice that the convexity term has improved the estimate provided by just the duration statistic, which would have been –\$7.771833. In general, duration alone overestimates the price fall when the yield rises and underestimates the price rise when the yield falls. Convexity, always a positive number on a fixed–income bond with no embedded options, improves the estimate by correcting for the curvature in the bond's price–yield relationship.

V. ANNUALIZED DURATION AND CONVEXITY

The previous numerical example states all variables in terms of the timing of the underlying cash flows, semiannual in that case. In particular, the duration and convexity statistics are calculated using the coupon rate and yield *per period,* and the number of *periods* to maturity. In practice, these statistics are typically "annualized." Let the annualized yield to maturity be denoted YLD, and the number of periods in the year PER, so that $YLD = PER \times i$ and $dYLD = PER \times di$. Substituting these into Equation 17 and rearranging terms provides the expression that relates the change in the annualized yield to the change in market value:

$$dMV_t \approx \left[-\left(\frac{DUR_t}{PER \times \left(1 + \dfrac{YLD}{PER}\right)} \times MV_t \right) \times (dYLD) \right] + \left[\frac{1}{2} \times \left(\frac{CONV_t}{(PER)^2} \times MV_t \right) \times (dYLD)^2 \right] \tag{23}$$

DUR_t divided by PER is the annualized Macaulay duration and that divided by 1 plus the periodic yield is the annualized modified duration. $CONV_t$ divided by the periodicity squared is the annualized convexity. These statistics are "annualized" in the sense of their corresponding to the annualized yield. When DUR_t is 24.946 based on two periods per year, the annualized duration of the bond is 12.473. Likewise, the convexity of 902.903 in terms of semiannual periods can be annualized to 225.726. Annualized duration and convexity statistics are useful since a fixed–income analyst typically works with market movements stated in terms of annualized yields. For example, the owner of the 8% Treasury due in November of 2021 more likely articulates risk as a potential 50–basis–point move in the annualized yield rather than as a 25–basis–point move in the yield per semiannual period. The value–at–risk assessment shown in Equation 22 can be duplicated using the annualized statistics:

$$dMV_t \approx \left[\left(\frac{12.473}{1.03} \times \$128.357067\right) \times (.0050)\right]$$

$$+ \left[\frac{1}{2} \times (225.726 \times \$128.357067) \times (.0050)^2\right] \tag{24}$$

$$= -\$7.771833 + \$0.362169$$

$$= -\$7.409664$$

VI. FINANCIAL CALCULATOR PROGRAMS

Exhibit 2 offers programs for duration and convexity that can be used with the SOLVE menu in the popular Hewlett–Packard 17BII and 19BII financial calculators. To correspond to the variables named in the bond pricing menu (BOND), the annual coupon rate is CPN% and the annual yield to maturity is YLD%, both entered on a percentage basis. Other inputs are the years to maturity (YRS) as of the last coupon date, the number of periods in the year (PER), and the fraction of the period elapsed since the last coupon payment (FRACT). This is the same fraction that was denoted t/T earlier. The outputs of the four programs are the annualized duration and convexity statistics on a coupon date 0 and on any between–coupon date t.

To test the programs, consider again the duration and convexity statistics for the 8 percent Treasury bond due November 15, 2021, that is priced to yield 6.00 percent on February 29, 1996. Inputting 2 PER, 6 YLD%, 26 YRS, and 8 CPN% gives the annualized duration (DUR,0) as 12.764 on the last coupon date. In the next program, entering $106 \div 182 = 0.582$ FRACT gives the annualized duration (DUR,T) on February 29 as 12.473. Note that the SOLVE menu allows common variables to be shared across equations so it is not necessary to re–enter the value for DUR,0. The annualized convexity on the last coupon date, denoted CON,0 in the program, would have been 232.790. In the last program, the value for February 29 (CON,T) is shown to be 225.726, the same as calculated earlier. Note that the first and third programs can be used to verify textbook duration and convexity examples, which typically are limited to just coupon dates.

Exhibit 2
Financial Calculator Programs for Duration and Convexity

Duration on a Coupon Date

DURATION, DATE 0: DUR,0 = ((100 × PER + YLD%) ÷ YLD%
- ((100 × PER) + YLD% + PER × YRS × (CPN% – YLD%))
÷ (CPN% × ((1 + YLD% ÷ (100 × PER)) ^ (PER × YRS))
– CPN% + YLD%)) ÷ PER

Duration Between Coupon Dates

DURATION, DATE T: DUR,T = (DUR,0 × PER – FRACT) ÷ PER

Convexity on a Coupon Date

CONVEXITY, DATE 0: CON,0 = (2 × CPN% × SQ (PER × 100
+ YLD%) × (((YLD% ÷ PER ÷ 100 + 1) ^ (YRS × PER))
– 1 – ((YLD% × YRS × PER) ÷ (PER × 100 + YLD%)))
+ ((YRS × PER) × (YRS × PER + 1) × SQ (YLD%)
× (YLD% – CPN%))) ÷ (SQ (YLD%) × SQ (YLD% ÷ PER
÷ 100 + 1) × (CPN% × (((YLD% ÷ PER ÷ 100 + 1) ^ (YRS
× PER)) – 1) + YLD%)) ÷ SQ (PER)

Convexity Between Coupon Dates

CONVEXITY, DATE T: CON,T = (CON,0 × SQ (PER) – FRACT
÷ SQ (YLD% ÷ 100 ÷ PER + 1) × (2 × DUR,0 × PER
+ (1 – FRACT))) ÷ SQ (PER)

VII. CONCLUSION

A few duration and convexity calculations done by direct substitution into the formulas or by entering the inputs into a program on a financial calculator serve to remind one of the value of computer spreadsheets. Yet working only with software packages that generate these statistics can let one forget the source of the formulas. This note demonstrates that the derivation for duration and convexity, even between coupon dates, involves nothing other than the Taylor's series expansion and basic calculus and algebra. Too often such derivations are left out of textbooks and training courses. Too often analysts view their spreadsheets as "black boxes." From time to time it can be worthwhile to review the derivations, if for no other reason than to remind oneself of the limitations of the statistics.

The key steps in the derivations are the first and second partial derivatives of the general price–yield relationship. The key result is an estimate of the change in market value of a bond given a change in its yield to maturity. But a number of different market events can generate the same change in the bond's yield. The duration and convexity statistics do not distinguish between a change in the benchmark Treasury bond and a change in the credit spread relative to that Treasury. Moreover, a variety of Treasury yield curve shifts and twists can lead to the same change in the bond's yield to maturity. These statistics do not isolate sensitivity to varying points along the yield curve, nor do they incorporate the covariance with other bonds' yields. In sum, duration and convexity are very useful starting points in measuring value at risk, but they are hardly the ending points.

APPENDIX A
Derivation of the Duration Statistic on a Coupon Date

Let the coupon rate per period, PMT/FV, be denoted c. Then, the closed–form expression for PV_0 in Equation 4 in the text can be rewritten as:

$$PV_0 = \left\{\left[\frac{c}{i} \times \left(1 - \frac{1}{(1+i)^N}\right)\right] + \frac{1}{(1+i)^N}\right\} \times FV \qquad \text{(A1)}$$

The first partial derivative of this with respect to the yield i is:

$$\begin{aligned}\frac{\partial PV_0}{\partial i} = \Bigg\{\left[\frac{c}{i} \times \frac{N}{(1+i)^{N+1}}\right] \\ - \left[\frac{c}{i^2} \times \left(1 - \frac{1}{(1+i)^N}\right)\right] \\ - \left[\frac{N}{(1+i)^{N+1}}\right]\Bigg\} \times FV\end{aligned} \qquad \text{(A2)}$$

Multiply this by $-(1+i)$ and divide by PV_0 as expressed in Equation A1 to get an explicit expression for DUR_0, as it has been defined in the text by Equation 8:

$$DUR_0 = \frac{-\left[\frac{c}{i} \times \frac{N}{(1+i)^N}\right] + \left[\left(\frac{c \times (1+i)}{i^2}\right) \times \left(1 - \frac{1}{(1+i)^N}\right)\right] + \left[\frac{N}{(1+i)^N}\right]}{\left[\frac{c}{i} \times \left(1 - \frac{1}{(1+i)^N}\right)\right] + \frac{1}{(1+i)^N}} \qquad \text{(A3)}$$

Next, simplify this expression by multiplying both the numerator and denominator by $[i \times (1+i)^N]$:

$$DUR_0 = \frac{-[c \times N] + \left[\left(\frac{c \times (1+i)}{i}\right) \times \left((1+i)^N - 1\right)\right] + [i \times N]}{\left[c \times \left((1+i)^N - 1\right)\right] + i} \quad \text{(A4)}$$

Next, add $(1 + i)/i) \times i$ and subtract $(1 + i)$ to the numerator in Equation A4 and rearrange terms:

$$DUR_0 = \frac{\left[\frac{1+i}{i} \times c \times \left((1+c)^N - 1\right)\right] + \left[\frac{1+i}{i} \times i\right] - (1+i) - (c \times N) + (i \times N)}{\left[c \times \left((1+i)^N - 1\right)\right] + i} \quad \text{(A5)}$$

Finally, simply terms to obtain:

$$DUR_0 = \frac{1+i}{i} - \frac{1 + i + [N \times (c - i)]}{\left[c \times \left((1+i)^N - 1\right)\right] + i} \quad \text{(A6)}$$

APPENDIX B

Derivation of the Convexity Statistic on a Coupon Date

The first partial derivative for a change in PV_0 to a change in the yield was derived in Equation A2 and repeated here:

$$\frac{\partial PV_0}{\partial i} = \left\{ \left[\frac{c}{i} \times \frac{N}{(1+i)^{N+1}} \right] - \left[\frac{c}{i^2} \times \left(1 - \frac{1}{(1+i)^N} \right) \right] - \left[\frac{N}{(1+i)^{N+1}} \right] \right\} \times FV \qquad \text{(B1)}$$

The second partial derivative is the following:

$$\frac{\partial^2 PV_0}{\partial i^2} = \left\{ - \left(\frac{c}{i} \times \frac{N \times (N+1)}{(1+i)^{N+2}} \right] - \left[\frac{2 \times c}{i^2} \times \frac{N}{(1+i)^{N+1}} \right] + \left[\frac{2 \times c}{i^3} \times \left(1 - \frac{1}{(1+i)^N} \right) \right] + \left[\frac{N \times (N+1)}{(1+i)^{N+2}} \right] \right\} \times FV \qquad \text{(B2)}$$

Divide this by PV_0, as expressed in Equation Al, and then multiply the numerator and denominator by $i^3 \times (1+i)^{N+2}$ and rearrange terms to obtain the closed–form equation for $CONV_0$:

$$CONV_0 = \frac{\left[2 \times c \times (1+i)^2 \times \left((1+i)^N - \frac{1+i+(i \times N)}{1+i} \right) \right] + \left[N \times (N+1) \times i^2 \times (i-c) \right]}{i^2 \times (1+i)^2 \times \left\{ \left[c \times \left((1+i)^N - 1 \right) \right] + i \right\}} \qquad \text{(B3)}$$

NOTES

1. The classic reference for duration is Macaulay (1938), Fabozzi (1996) is an excellent source for applications of duration and convexity in investment analysis. Authors providing closed-form formulas for duration and convexity include Chua (1984, 1988); Caks, Lane, Greenleaf, and Joules (1985); Smith (1988); Moser and Lindley (1989); Taylor (1989); Brooks and Livingston (1989); Nawalka, Lacey, and Schneeweis (1990); Lacey and Nawalkha (1990); Nawalkha and Lacey (1991); and Blake and Orszag (1996).

2. See Whitmore (1985) and Smith (1997) for derivations of an expression for accrued interest that includes the time value of money.

REFERENCES

Blake, D. and J. M. Orszag, "A Closed–Form Formula for Calculating Bond Convexity," *The Journal of Fixed Income,* 6, June 1996, pp. 88–91.

Brooks, R. and M. Livingston, "A Closed–Form Equation for Bond Convexity," *Financial Analysts Journal,* 45, November/December 1989, pp. 78–9.

Caks, J., W. R. Lane, R. W. Greenleaf, and F. G. Joules, "A Simple Formula for Duration," *Journal of Financial Research,* 8, Fall 1985, pp. 245–9.

Chua, J. H., "A Closed–Form Formula for Calculating Bond Duration," *Financial Analysts Journal,* 40, May–June 1984, pp. 76–8.

Chua, J. H., "A Generalized Formula for Calculating Bond Duration," *Financial Analysts Journal,* 44, September/October 1988, pp. 65–7.

Fabozzi, F. J., *Bond Markets, Analysis, and Strategies,* 3e, Upper Saddle River, NJ: Prentice Hall, 1996.

Lacey, N. J. and S. K. Nawalkha, *Closed–Form Duration Measures and Strategy Applications*, Charlottesville, VA: The Research Foundation of The Chartered Financial Analysts, 1990.

Macaulay, F. R., *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the U.S. since 1856*, New York: National Bureau of Economic Research, 1938.

Moser, J. T. and J. T. Lindley, "A Simple Formula for Duration: An Extension," *The Financial Review,* 24, November 1989, pp. 611–15.

Nawalkha, S. K. and N. J. Lacey, "Convexity for Bonds with Special Cash Flow Streams," *Financial Analysts Journal,* 47, January–February 1991, pp. 80–2.

Nawalkha, S. K., N. J. Lacey, and T. Schneeweis, "Closed–Form Solutions of Convexity and M–Square," *Financial Analysts Journal,* 46, January–February 1990, pp. 75–7.

Smith, D. J., "The Duration of a Bond as a Price Elasticity and a Fulcrum," *Journal of Financial Education,* 17, Fall 1988, pp. 26–38.

Smith, D. J., "Bond Pricing Between Coupon Dates and Theoretically Correct Accrued Interest," Manuscript, September 1997.

Taylor, R. W., "Bond Duration with Geometric Mean Returns," *Financial Analysts Journal,* 45, January/February 1989, pp. 77–80.

Whitmore, G. A., "A Note on the Exact Calculation of Accrued Interest and Bond Pricing," *Financial Analysts Journal,* 41, March–April 1985, pp. 76–7.

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