Functional Data Analysis

Lecture – 1

Course contents and the textbook

Functional Data Analysis with R and MATLAB by J. O. Ramsay, G. Hooker, and S. Graves; Springer; 2009.

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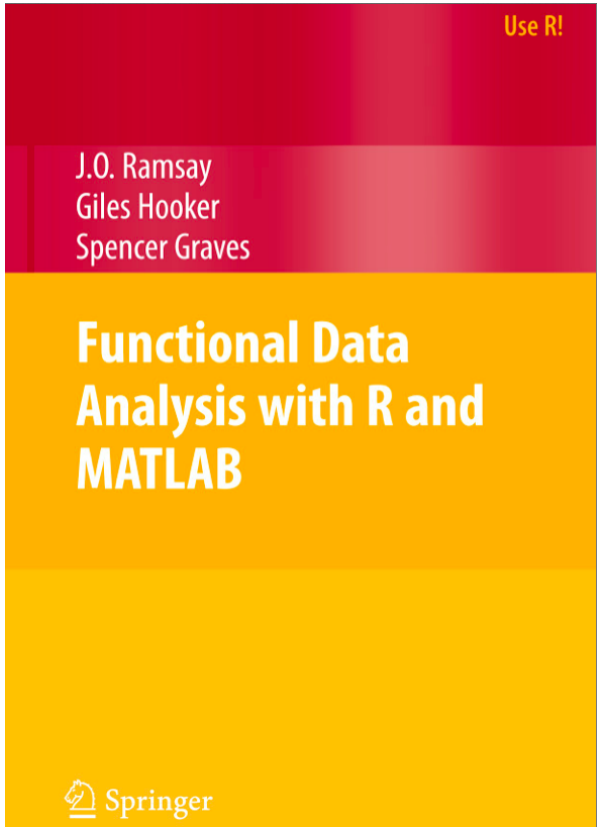

- Provides a complete overview of the subject together with a healthy mix of examples.
- Also provides help with `R` whenever needed.

Reference for `R` package on FDA:

`https://cran.r-project.org/web/packages/fda/index.html`

Access to the textbook

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- All the datasets included in the textbook come along with the `fda` package.

Outline

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- The choice is largely dictated by the decision to stick to the textbook.

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- There some so-called R front-ends (such *R Commander* or *R-Studio* or *Jupyter*) that ease writing more complex programming in R – while you can use and utilize them, I assume **only a very basic R installation** with the primitive *copy-and-paste-to-the-command-line* approach as a method of running the programs.

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"Data providing information about curves, surfaces or anything else varying over a continuum. In its most general form, under an FDA framework each sample element is considered to be a function. The physical continuum over which these functions are defined is often time, but may also be spatial location, wavelength, probability, etc."

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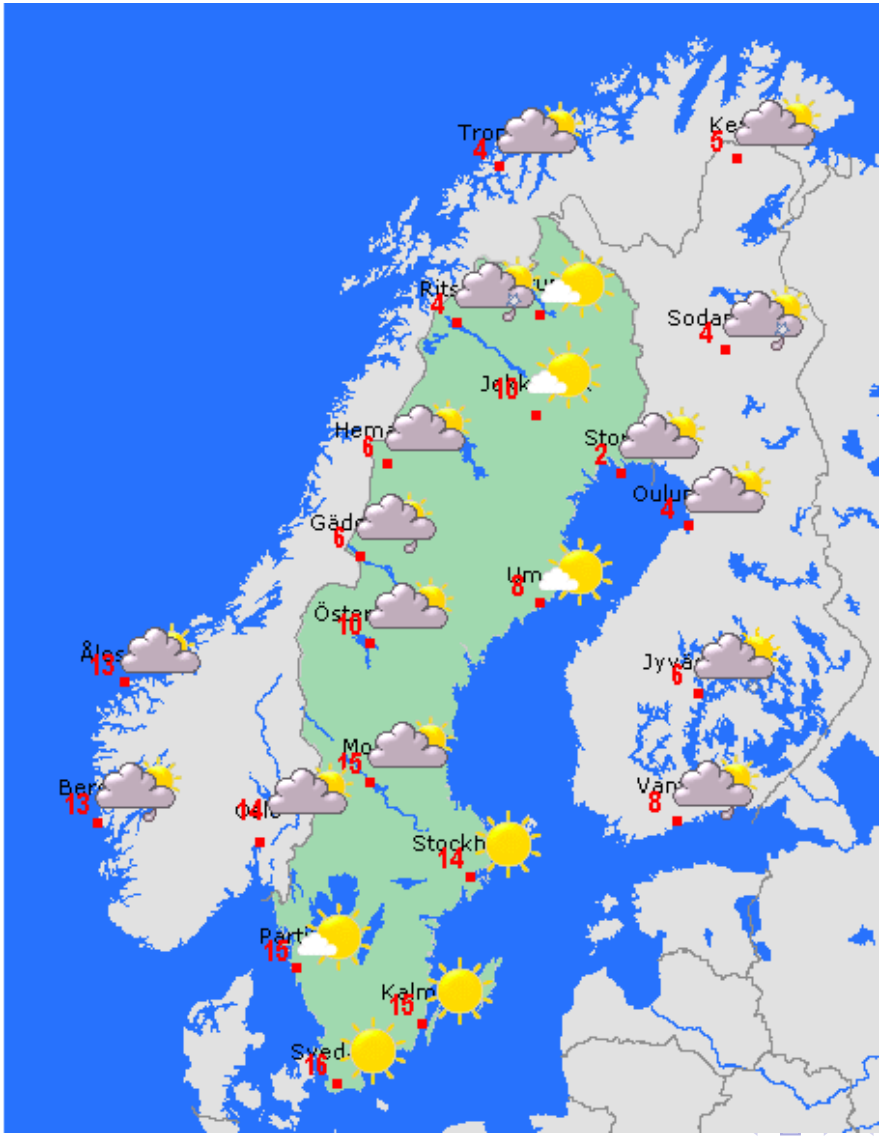
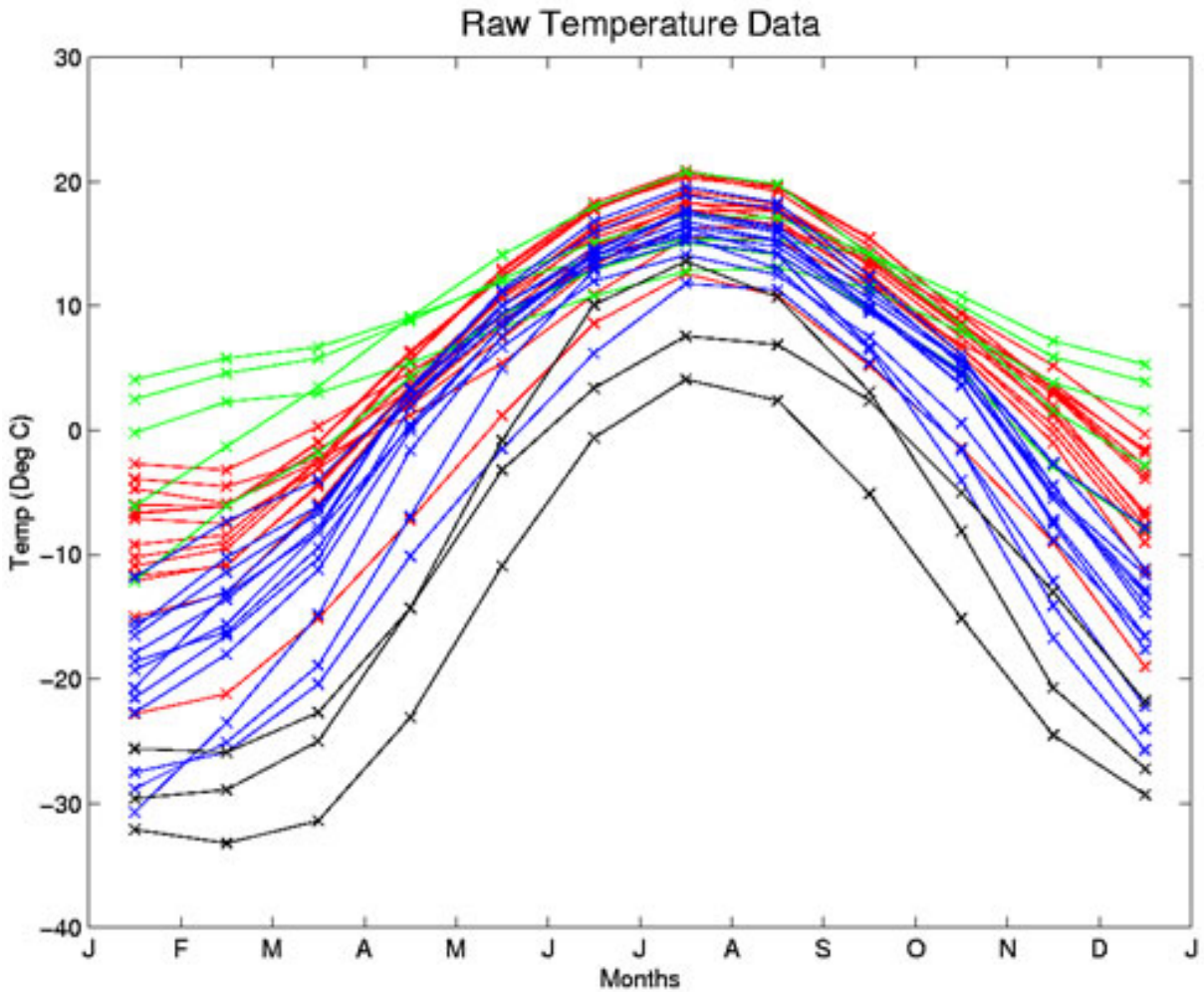
Examples: Temperature over a Sweden– spatial



Examples: Temperature at a location over a given period of time



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- How do the shapes of the weather patterns differ among the Pacific, Continental, Atlantic, and Arctic climates? How does the weather pattern in say, Lund, differ from the typical pattern in the rest of the Scandinavian?

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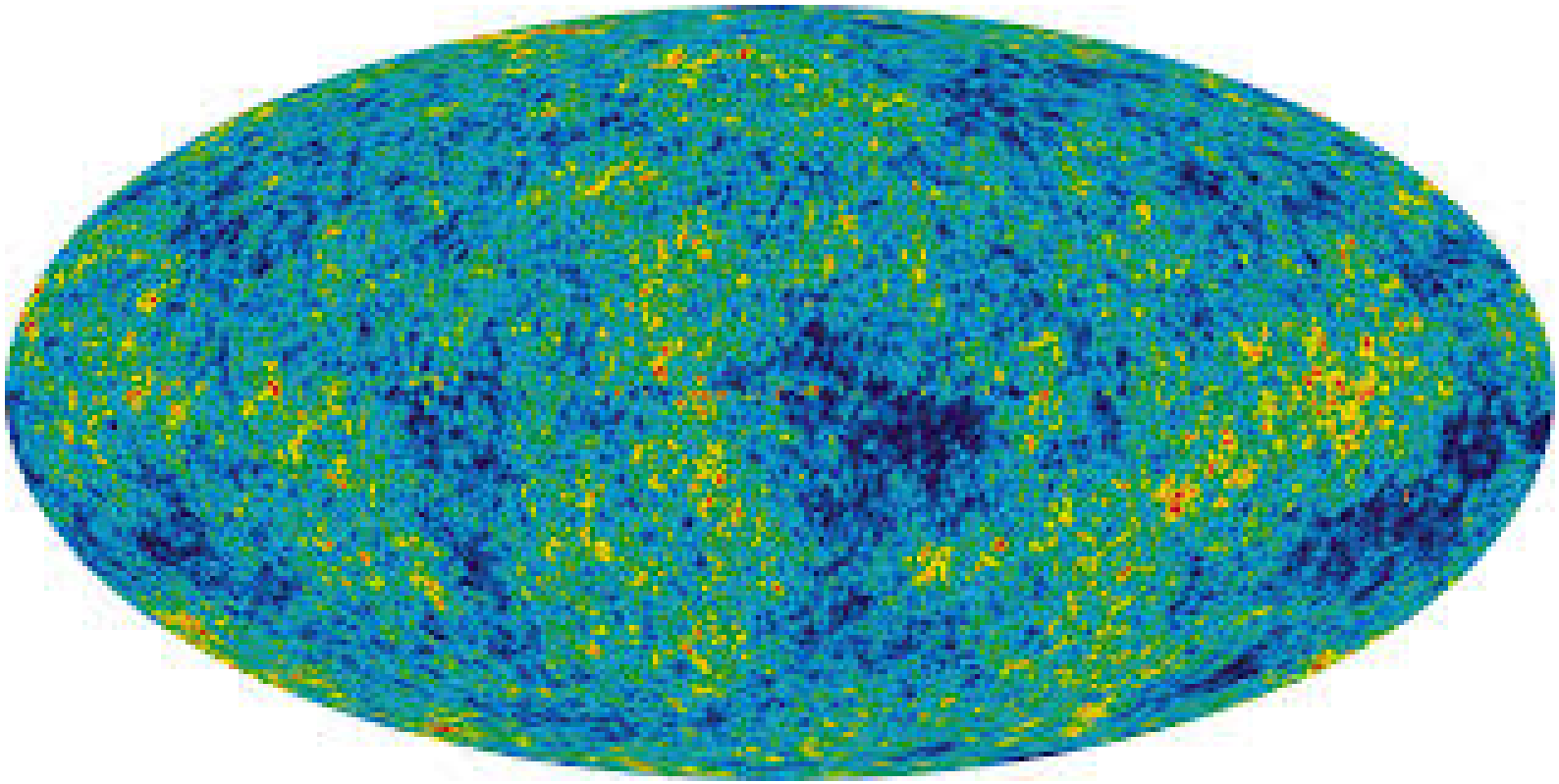
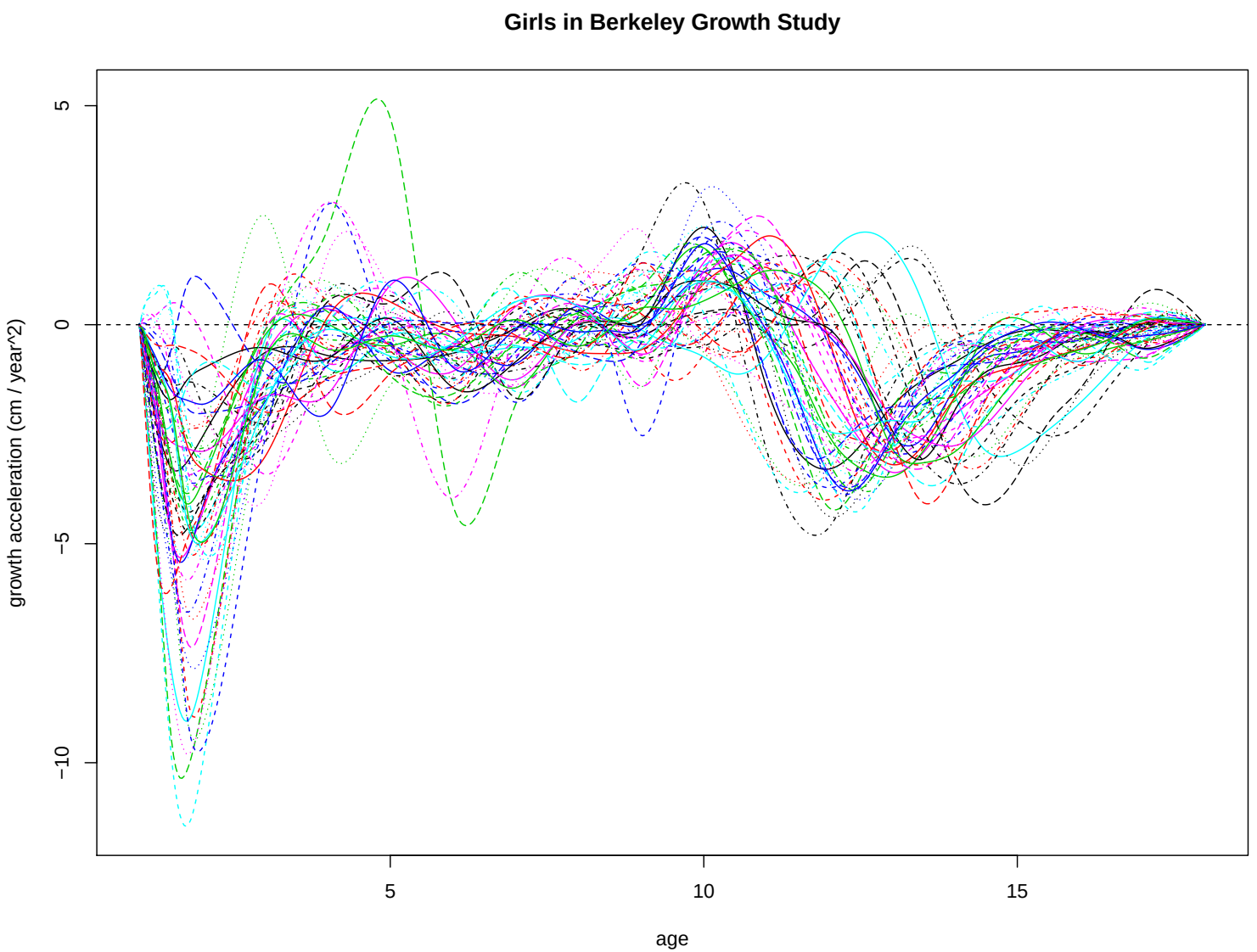
- How can we determine the primary modes of variation in the data? How many typical modes can summarize these thirty-five curves?
- Do these curves exhibit strictly sinusoidal behavior?
- Can we create an analysis of variance (ANOVA) or linear model with the curves as the response and the climate as the main effect?

Examples: Cosmic microwave background radiation

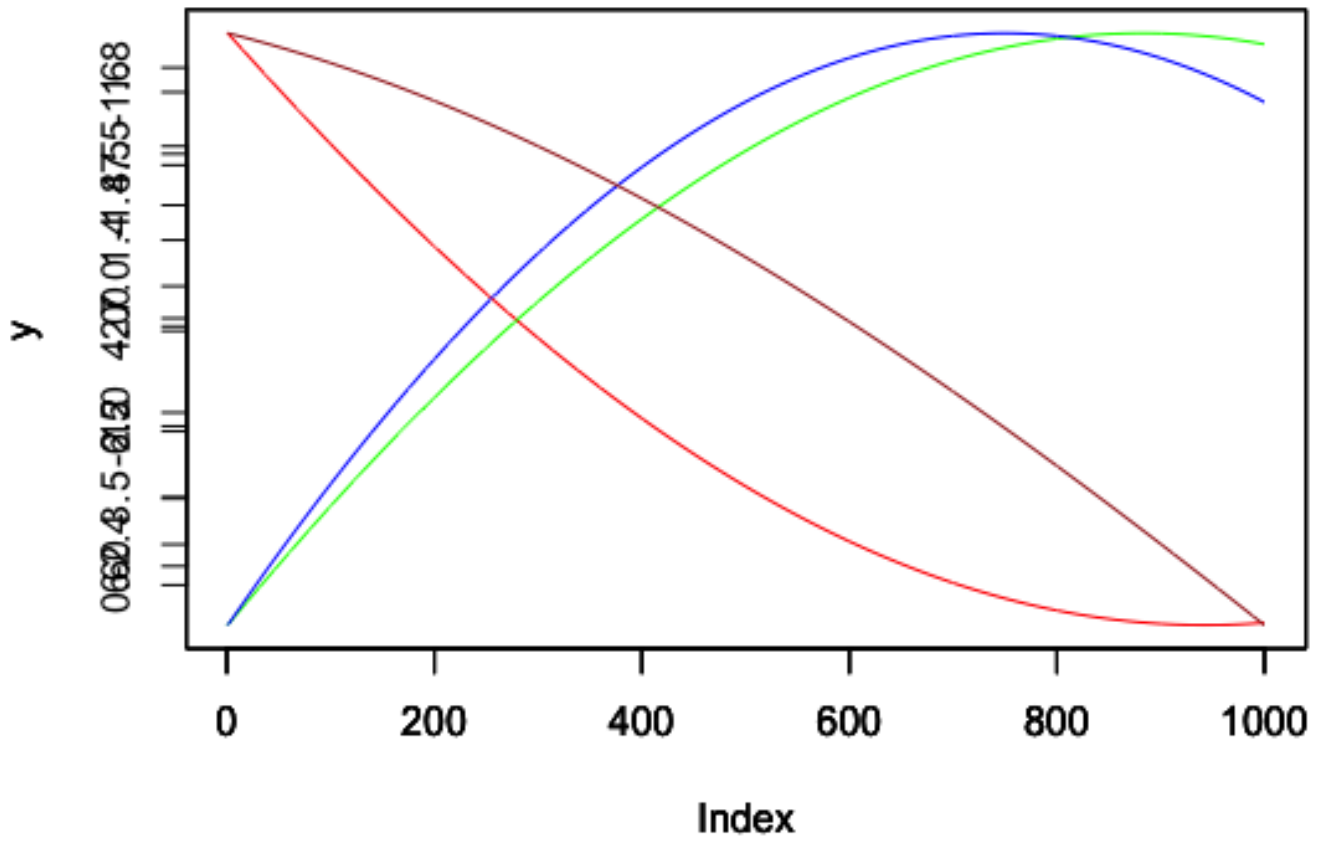


Interest is primarily modeling, and extracting features / patterns.

Examples: Growth data of girls



A fun example: finite dimensional functional data



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The figure corresponds to four realizations of

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NOTE: Brownian motion can be represented as an infinite series of the above kind. Thus, most of the physical processes we wish to model/analyse are infinite dimensional in nature.

Objective of FDA

Our interest is in...

- Representations of distribution of functions
  - mean
  - variation
  - covariation
- Relationships of functional data to
  - covariates
  - responses
  - other functions
- Relationships between derivatives of functions.
- Timing of events in functions.

... and the challenges are...

- Estimation of functional data from noisy, discrete observations.
- Numerical representation of infinite–dimensional objects
- Representation of variation in infinite dimensions.
- Description of statistical relationships between infinite dimensional objects.
- In case the covariates are larger than the observations? (regularisation and smoothness)
- Measures of variation and confidence in estimates.

Concluding thought

We are drowning in information and starving for knowledge

Rutherford D. Roger*

*American librarian (Yale University)